Enhanced - resolution material imaging with dielectric resonators: a new implicit space - domain technique

Malgorzata Celuch#1, Wojciech Gwarek#2, Andrzej Wieckowski#3
#QWED Sp. z o.o., Poland
1mceluch@qwed.eu, 2wgwarek@qwed.eu, 3awieckowski@qwed.eu

***Abstract*—A new method of resolution improvement for dielectric resonator material measurements is proposed. Initially, a material sample is scanned with the resonator over a 2D mesh of scanning points, and thereby at each point a weighted average of complex permittivity over the region interacting with the resonator fields is produced. Then a space-domain implicit (SDI) problem is formulated that relates the explicit measurements to the enhanced permittivity pattern through the pre-simulated electric field pattern of the resonator. A robust SVD-based technique for solving the implicit problem is developed. The SDI method is validated on virtual samples and successfully applied to the available laboratory scan.**

***Keywords*—material measurements, non-destructive testing, dielectric resonators, spatial resolution, FDTD.**

I. Motivation of the Work

Dielectric resonator (DR) methods are an accurate and non-destructive techniques of material measurements at microwave frequencies. In particular, the split-post DR (SPDR) configuration [1] has been accepted as a popular standard for characterising low-loss laminar dielectrics. The single-post DR (SiPDR) configuration [2] has been developed for the measurements of the surface impedance of resistive films as well as for the contact-less conductivity measurements of semiconductor wafers. Both SPDR and SiPDR have been applied to metamaterials [3]. In both configurations, samples-under-test (SUT) are easily inserted without any dismantling of the resonator and material parameters are explicitly calculated from the measured resonant frequencies and Q-factors.

A limitation of DR measurements is, that they are essentially dedicated to homogeneous samples. To further allow the imaging material inhomogeneities, for example due to process variations in the production of semiconductors, the use of motorised scanners incorporating a DR has been reported [4],[5]. However, when the measurement at each scanning point is explicitly converted into the material parameters, the result is not a punctual value of the parameter, but rather its specific average over the area interacting with the DR field. In other words, the spatial resolution of standard DR measurements is limited by the DR head size.

An SPDR with a nominal frequency of 10 GHz provides complex permittivity of the sample averaged over ca. 16 mm diameter. While theoretically the spatial resolution increases linearly with the increase of frequency (and smaller DR head), such an approach is impractical due to the associated restrictions on the SUT thickness and mechanical manufacturing tolerances. The family of SPDRs readily available on the market terminates at 15 GHz with ca. 12 mm effective resolution. This stimulates the interest in alternative techniques for resolution enhancement for DR measurements, to which our work responds, as follows:

- In Section II we summarise the principles of standard explicit DR measurements and explain how the resolution can be improved with the use of pre-simulated electric field pattern in the active space of the resonator, for example those produced by the finite-difference time-domain (FDTD) method [6],[7]. The space-domain implicit (SDI) measurement technique is thereby formulated.

- In Section II we implement the DR-SDI technique in MATLAB environment and validate it in application to virtual samples and measurements. We propose a methodology leading to robust solutions in the presence of practical levels of measurement noise.

- In Section III the DR-SDI technique is applied to a laboratory scan of a real-life dielectric sample and in Section IV - to a big inhomogeneous sample.

In the Conclusions we indicate further developments of the DR-SDI method and its planned applications, including a systematic comparison to the recently proposed implicit technique based on deconvolution in the Fourier domain [8].

II. Implicit versus Explicit DR Measurements

Measurements of complex permittivity in DRs are based on the fundamental relationships between the resonant frequency and Q-factor, on the one hand - and the electric energy stored and power dissipated in the resonator, on the other hand. When a SUT of relative permittivity $\varepsilon_s = \varepsilon_s' - j\,\varepsilon_s''$ is inserted into the resonator, the resonant frequency changes from f_e to f_s and the Q-factor changes from Q_e to Q_s. Since DR measurements are taken in the tangential electric field [1],[3] and dedicated to thin planar samples [1]-[3], both the field and the permittivity can be assumed constant along height h of the sample, leading to:

$$\frac{f_e - f_s}{f_e} \approx \frac{h}{2C} \iint_S \left[\varepsilon_s'(x,y) - 1\right] \left|E(x,y)\right|^2 dS \quad (1)$$

$$\frac{1}{Q_s} - \frac{1}{Q_e} \approx \frac{h}{C} \iint_S \varepsilon_s''(x,y) E^2(x,y) dS \quad (2)$$

$$C = h\iint_S \left|E(x,y)\right|^2 dS \tag{3}$$

where the surface of integration S is called the DR's *head*. The approximation in Eqs. (1) and (2) reflects changes of E-field caused by the sample, negligible for low-permittivity materials.

When SUT is homogeneous and large enough to cover the DR's head (16 mm x 16 mm for the 10 GHz SPDR, see Fig. 1), one measurement of the empty resonator followed by one measurement of the loaded resonator allows *explicitly* extracting the real and imaginary parts of permittivity by Eqs. (1) and (2), respectively. Further in this summary we focus on the extraction of ε_s' through eq. (1), though analogous considerations apply to ε_s". In the standard SPDR method [1] ε_s' is calculated iteratively by formula:

$$\varepsilon_s' = 1 + \frac{f_e - f_s}{h f_e} \frac{1}{K\left(\varepsilon_s', h\right)} \tag{4}$$

where function K reflecting small E-field changes by the sample is pre-computed (in [1], by the Rayleigh-Ritz method) for different values of h and ε_s', and tabulated. Function K is slowly-varying and hence a few iterations over ε_s' converge rapidly to its final value.

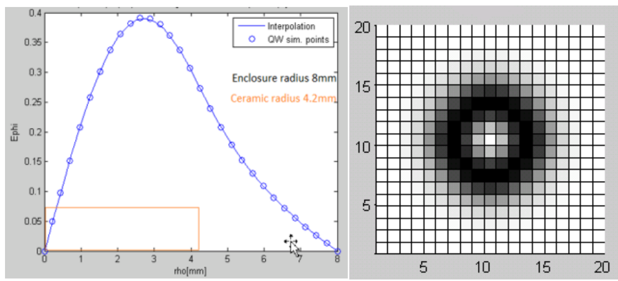

Fig. 1. E-field pattern of 10 GHz SPDR: left - radial distribution of azimuthal E-field simulated with BOR FDTD [7]; right - SPDR template (squared E-field extrapolated in MATLAB onto 1 mm mesh over the SPDR head).

When SUT is inhomogeneous or comparable to the head size, the measurement following the above standard *explicit* procedure provides only the averaged values of permittivity. In [4],[5], DR images of inhomogeneous semiconductor surfaces have been reported. The 5 GHz SiPDR has been built into a mechanical scanner and the explicit measurement procedure has been applied at each position. Surface resistivity maps have thereby been obtained, however, as explained above, the value assigned to each point is actually an average taken over the DR's head. The averaging (weighting) function is the squared E-field distribution, which we shall further call the DR's *template*. In other words, resolution of explicit DR measurements is limited by the size of the head, and smaller-than-head SUTs are "dissolved" in the background medium, as we demonstrate in further examples.

We now introduce a method to enhance the resolution of DR measurements without the need for reducing the size of the head (and thus increasing the operating frequency). We take advantage of the fact that the DR template $E_T(x,y)$ is known from EM simulations. The Rayleigh-Ritz method as in [1] is one possible choice, but here we apply a general purpose BoR (bodies-of-revolution) FDTD method [6] implemented in QuickWave software [7]. As an example, a modal E-field pattern under the head of the 10 GHz SPDR is shown in Fig. 1.

We take a sequence of DR measurements at the mesh of M x N points over the sample, as in [4],[5], but rather than applying the explicit conversion of measured f_s to ε_s' with formulae such as (4), we construct an array of measured $f_{s,mn}$ (or alternatively, measured energy changes ΔW_{mn} over the head) related to an array of unknown $\varepsilon_{s,mn}$ values. Consider the head meshed into (2K +1) x (2L+1) cells (in Fig. 1, K=L=8) whose center with $E_T(0,0)$ is placed at cell (m,n) of the scan. For clarity, assume that the mesh is equidistant of raster a (a = 1mm in Fig. 1). The measured energy change due to the SUT is:

$$\Delta W_{mn} = \frac{a^2 h}{2} \sum_{k=-K}^{K} \sum_{l=-L}^{L} \left[\varepsilon_s'(m+k, n+l) - 1\right] E_T^2(k,l) \tag{5}$$

Arranging the 2D array of ΔW_{mn} into a 1D vector W of elements ΔW_i, i=(n-1)*M+m, i=1,..,M*N, and similarly the 2D array of permittivities $p_{s,mn}=(\varepsilon_s'-1)_{mn}$ into vector P, we obtain an *implicit* formulation for the resolved permittivity image:

$$[W] = [T]\,[P] \tag{6}$$

Matrix T is generated in such a way that element t_{rs} in row r and column s is equal to :
- $|E_T(k, l)|^2$ for $s= r+k+M\,l$ for $k=-K..+K$ and $l=-L\ldots+L$
- 0 for s not obeying the above condition.

The resolved permittivity image is obtained by solving an inverse problem:

$$[P] = [T]^{-1}\,[W] \tag{7}$$

Note that the system matrix T is large, $M*N$ x $M*N$, but sparse. It also has a characteristic banded structure. We therefore expect to solve the implicit DR measurement problem (6)(7), producing the resolved permittivity image, with general-purpose inversion techniques for sparse and banded matrices, such as those available in MATLAB.

Admittedly, a different implicit technique for resolution enhancement of DR images has recently been proposed in [8]. In [8] the measured 2D pattern of energy changes ΔW_{mn} and the unknown pattern of permittivites $p_{s,mn}$ as defined above are Fourier - transformed. Then an implicit formulation analogous to eq. (6) is considered in the Fourier (spectral) domain, where it has the mathematical sense of deconvolution. To distinguish between the two implicit DR methods, we shall call ours Space-Domain Implicit (DR-SDI) and that of [8] Fourier-Domain Implicit (DR-FDI). Comparison of DR-SDI and DR-FDI is planned in future works.

III. Validation of DR-SDI Method on Virtual Samples

Consider a virtual scan of a 50 mm x 50 mm region presented in Fig. 2. The region includes a 20 mm x 20 mm square and a 1 mm dot, both of ε_s'=3, placed in air. In Fig. 2 (and further on) the upper row shows 2D permittivity distribution while the lower rows - horizontal (y=const.) and vertical (x=const.) cuts. For consistency with our subsequent real-life measurements, we assume that the virtual measurement is conducted with the real-life 10 GHz SPDR, having the template of Fig. 1.

The second column of Fig. 2 shows the results of our virtual measurement, obtained by scanning the template of Fig. 1 over the pattern of Fig. 2, left, in 1 mm steps. The map of energy changes ΔW_{mn} is shown in the 2nd column of Fig. 2. In explicit DR scanning, as in [4],[5], this energy map will proportionally scale to the permittivity map via eq. (4). The correct values of ε_s=3 and ε_s=1 are then obtained only at some points: when the head is either completely inside or completely outside of the sample bounds. For other positions, the material partially interacts with the field, the sample edges are diluted and the dot practically disappears.

The right column of Fig. 2 shows the SDI result. Not only the square sample, but also the small dot ($\frac{1}{16}$ of the DR head size) is reconstructed.

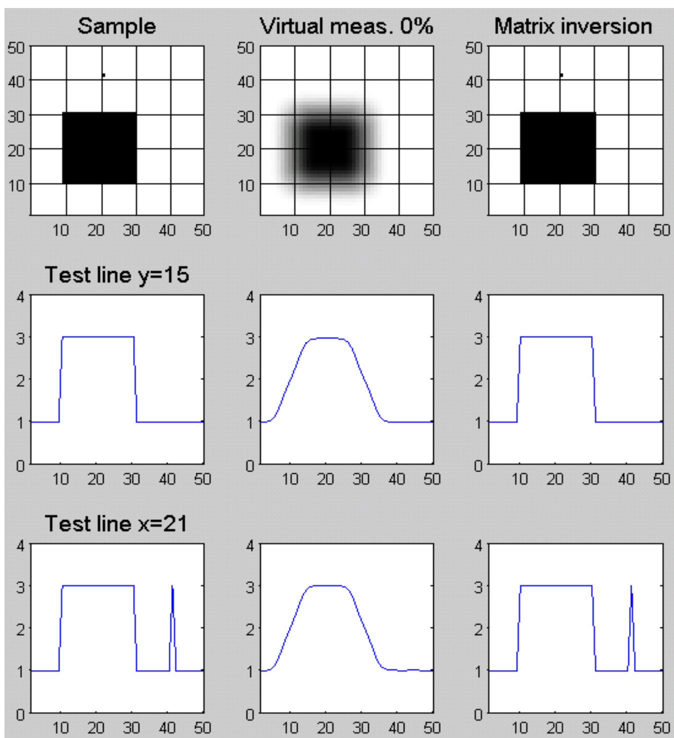

Fig. 2. Virtual measurement (with no added noise) of 20 mm x 20 mm sample and 1 mm dot on 50 mm x 50 mm background - and image reconstruction in MATLAB environment.

Having proven the SDI concept under the idealised conditions of noise-less measurements, we further add a random noise of 0.02% to each measurement (Fig. 3). In the virtual scan of the 1st column, this noise is hardly visible, but after matrix inversion it is drastically amplified and the result of the 2nd column hardly resembles the original. After standard noise filtering techniques, such as Wiener, the image is improved (3rd column), but far from being satisfactory.

The reason for noise amplification resides in the inherently very small determinant of the inverted matrix T. Thus, we have applied the technique of matrix regularisation. We perform Singular Value Decomposition (SVD) of matrix T and remove (set to zero) the smallest eigenvalues. We have found experimentally that retaining 10% eigenvalues typically leads to desired permittivity images. This is exemplified by the pattern in the 2nd column of Fig. 4, which restores the sharp edges and indicates the sample dot, though there remains visible noise over the sample area. Then the Wiener filter of size 8 averages out the noise, but also dilutes the dot, which indicates the need for further optimisation of the SDI method parameters.

The SVD approach works equally well in the presence of realistic laboratory noise, expected to be better than the level of 2% in SPDR measurements. This is shown in Fig. 5.

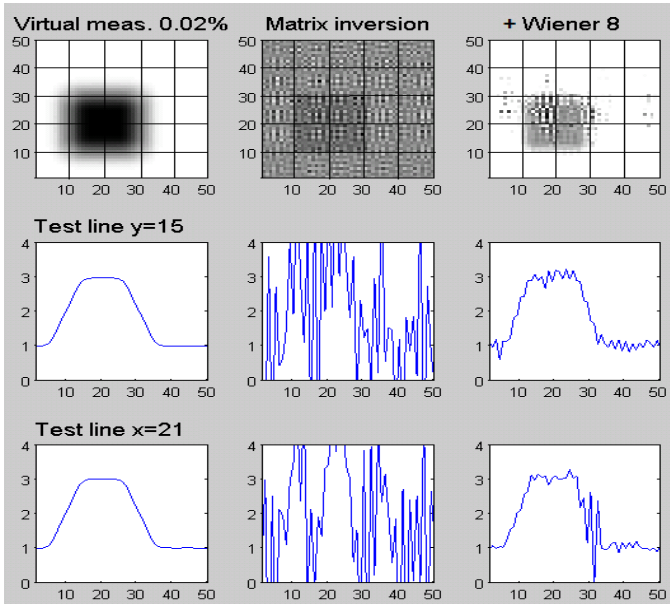

Fig. 3. Virtual measurement as in Fig. 2 but with random 0.02% noise added -and image reconstruction in MATLAB environment.

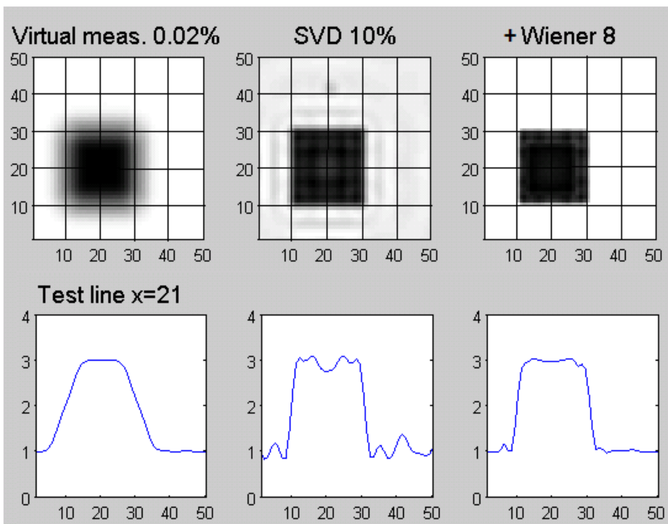

Fig. 4. Virtual experiment as in Fig. 3 but applying matrix regularisation with 10% eigenvalues retained.

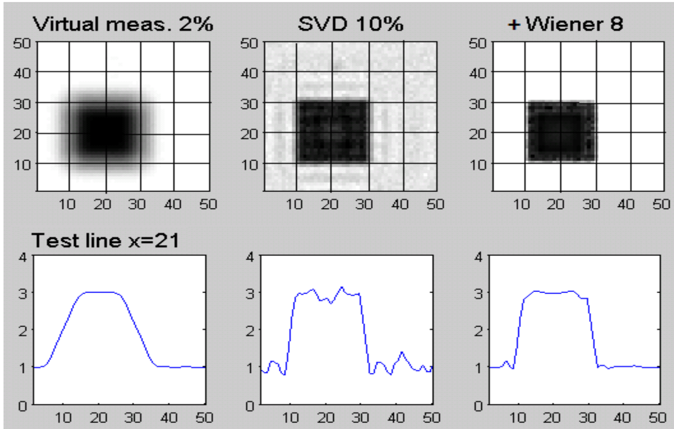

Fig. 5. Virtual experiment as in Fig. 4 but with random noise increased to 2%.

IV. Application of DR-SDI Method to a Real scan

Our real-life sample is a square piece of a laminate on a transparency so thin that it behaves like air inside the resonator. Hence this sample essentially mimics the virtual sample of Figs. 2-5, except for not having the small material dot. The sample has been scanned in 1 mm steps with the newly developed in-house 2D scanner, incorporating the same 10 GHz SPDR for which the template is shown in Fig. 1

The 2nd column of Fig. 6 shows the real explicit SPDR measurement. The results resolved by SVD are shown in the 3rd column, and Wiener filtering leads to the 4th column. As in the case of earlier virtual measurements, retaining 10% eigenvalues has been the best compromise for restoring the sharp sample edges without amplifying the (now actual) measurement noise.

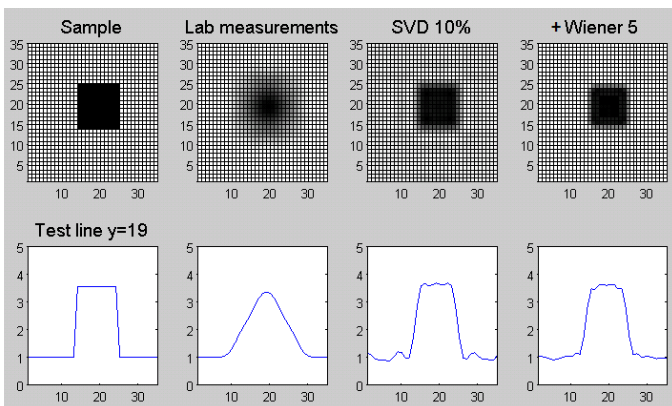

Fig. 6. MATLAB displays demonstrating the application of the SDI method to the permittivity image from the laboratory scanner with 10 GHz SPDR.

V. Application of the Method to Continuous Maps

In Fig. 7 we consider a virtual sample (blue curve), which however reflects a realistic variation of the material parameter continuously between the values of 8.5 and 10.5 (taken from [4]). The virtual explicit DR measurement (red) damps the variation, due to averaging under the head. Our DR-SDI method (violet) restores the maxima (10.5) and minima (8.5); further noise filtering (green) has a minor effect for this continuous pattern.

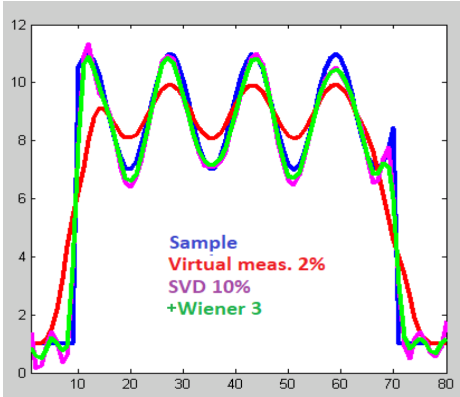

Fig. 7. MATLAB displays demonstrating the application of the SDI method to SUT following a realistic 1D variation of the measured parameter.

This example has been computationally expensive, in comparison to the examples of previous Sections. A scanning region in Figs. 2-5 was 50 x 50 points, while here it is 100 x 100. The SVD result of Figs. 2-5 was produced in 21 sec, while now 22 min are needed. This is consistent with the SVD scaling as N^6 known in linear algebra.

VI. Conclusions

A new Space-Domain Implicit method for resolution improvement of dielectric resonator material measurements has been proposed. The resonator is scanned over the material surface, but rather than *explicitly* converting the measurement at each step into permittivity with standard DR formulae, we combine all measurements into an *implicit* problem, where the system matrix is constructed from pre-simulated electric field pattern in the resonator. The matrix is ill-conditioned, but in the presence of practical measurement noise the implicit problem is still solved effectively by Singular Value Decomposition technique retaining ca. 10% of the highest eigenvalues, followed by noise filtering. The DR-SDI technique has been applied to down-scale a continuous permittivity image and to reconstruct samples of size comparable with the DR head. Resolution enhancement has been specifically demonstrated by detecting a dot of only $\frac{1}{16}$ of the DR head size. Our current research is concerned with optimising the parameters of SVD and noise filters, and SVD acceleration for larger samples (or at a higher scanning resolution) by domain decomposition.

Acknowledgment

This project has received funding from the European Union's Horizon 2020 research and innovation programme (H2020-NMBP-07-2017) under grant agreement MMAMA No. 761036.

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