



2022

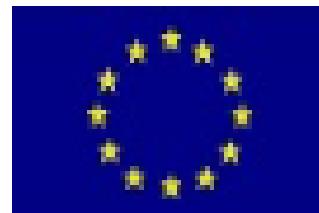
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# Modelling of CPW Structures for the Characterisation of Thin Film Materials

Konrad Wilczynski,<sup>\$</sup> Małgorzata Celuch,<sup>#</sup> Marzena Olszewska – Placha<sup>#</sup>

<sup>\$</sup>Faculty of Physics, Warsaw University of Technology, Poland

<sup>#</sup> QWED Sp. z o. o., Poland



Foundation for  
Polish Science

Presenter: Lukasz Nowicki, QWED Sp. z o. o., Poland



06.07.2022, Limoges

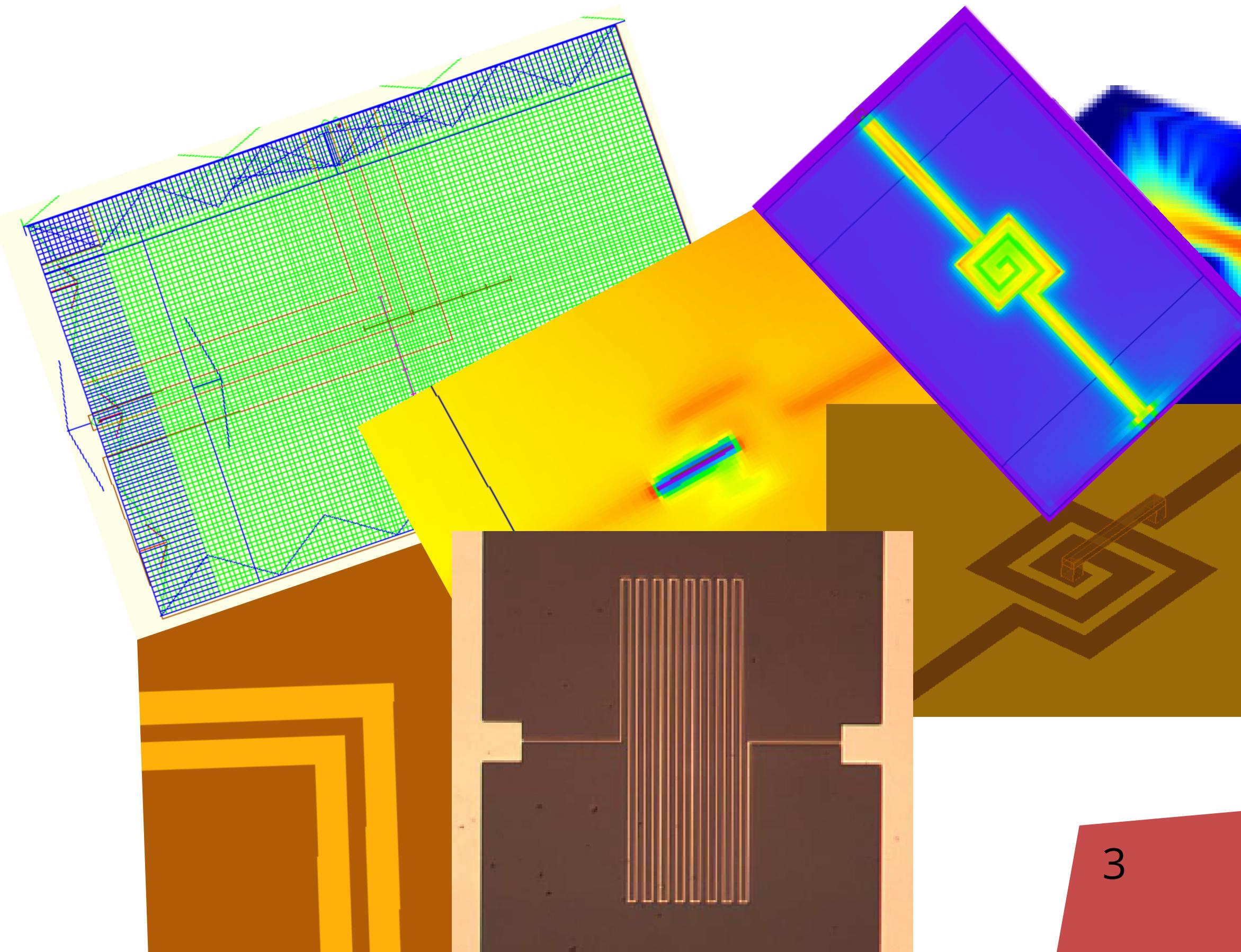
# Outline

- Coplanar waveguide structure **CPW**
  - Introducing the basics
  - Which EM parameters meaningfully describe thin film interaction with GHz waves?
- **CPW** modelling with the Finite Difference Time Domain **FDTD** method
  - How does one efficiently model a layer of ultra-small thickness in general purpose electromagnetic **EM** simulators?
- Application of the model to development of a 10 GHz dielectric resonator **DR** scanner for graphene anodes.
- Conclusions

# Coplanar waveguide structure CPW

## Introduction

Coplanar waveguides are a well-known variety of transmission lines used for example, to propagate of electrical signals with frequencies of the order of gigahertz (microwave range). Due to their simple structure, CPW waveguides are commonly used as elements of integrated microwave circuits.



## Complex dielectric function

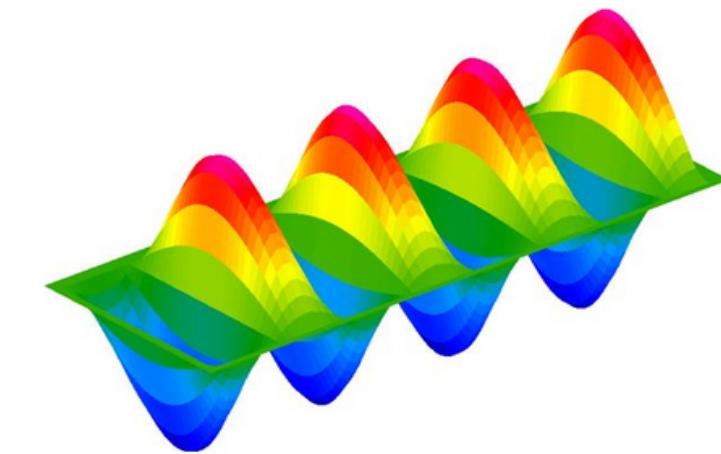
$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$



$$X(t) = \tilde{X}(\omega) \cdot e^{j\omega t}$$

$$\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}, \quad \nabla \cdot \tilde{\mathbf{B}} = 0, \quad \nabla \times \tilde{\mathbf{E}} = -j\omega \tilde{\mathbf{B}}, \quad \nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega \tilde{\mathbf{D}}$$

Maxwell's equations



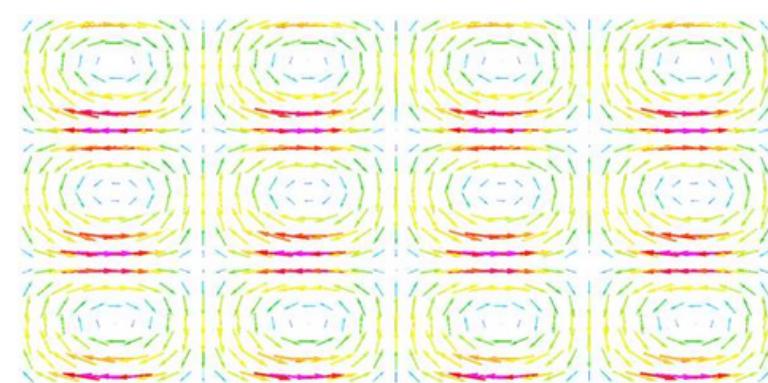
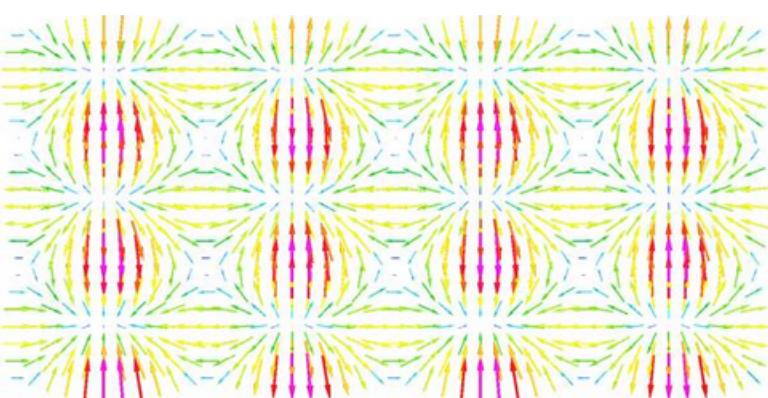
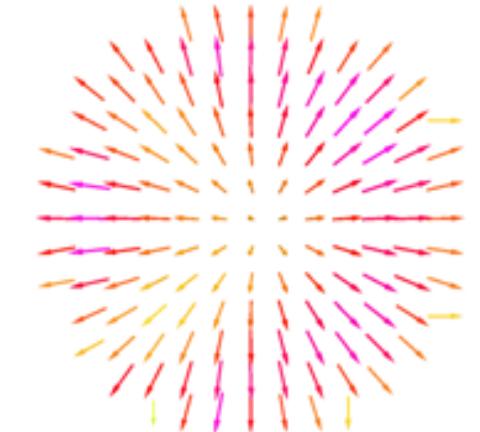
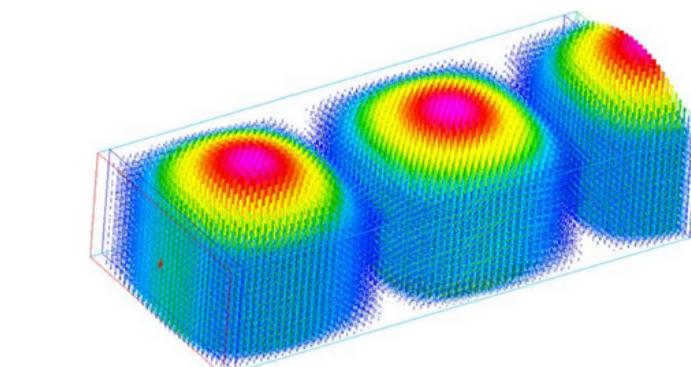
$$\tilde{\mathbf{D}} = \epsilon_0 \tilde{\epsilon}_r \cdot \tilde{\mathbf{E}}, \quad \tilde{\mathbf{B}} = \mu_0 \tilde{\mu}_r \cdot \tilde{\mathbf{H}}, \quad \tilde{\mathbf{J}} = \tilde{\sigma} \cdot \tilde{\mathbf{E}}$$

Constitutive equations in EM

$$\nabla \times \frac{\nabla \times \tilde{\mathbf{E}}}{\tilde{\mu}_r} - \omega^2 \mu_0 \epsilon_0 \left( \tilde{\epsilon}_r - j \frac{\tilde{\sigma}}{\omega \epsilon_0} \right) \cdot \tilde{\mathbf{E}} = 0, \quad \tilde{\mathbf{H}} = \frac{-1}{j\omega \mu_0 \tilde{\mu}_r} \nabla \times \tilde{\mathbf{E}}$$

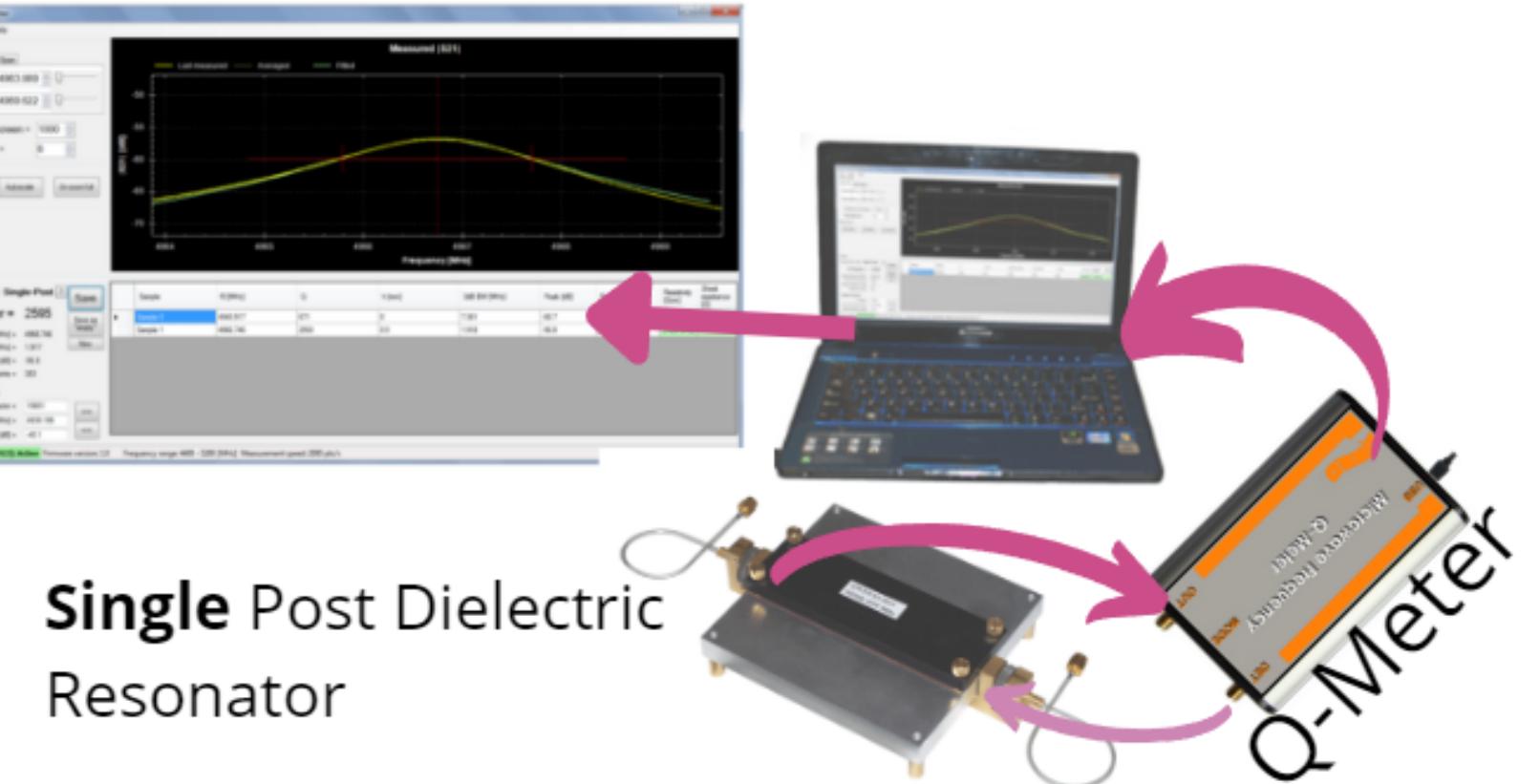
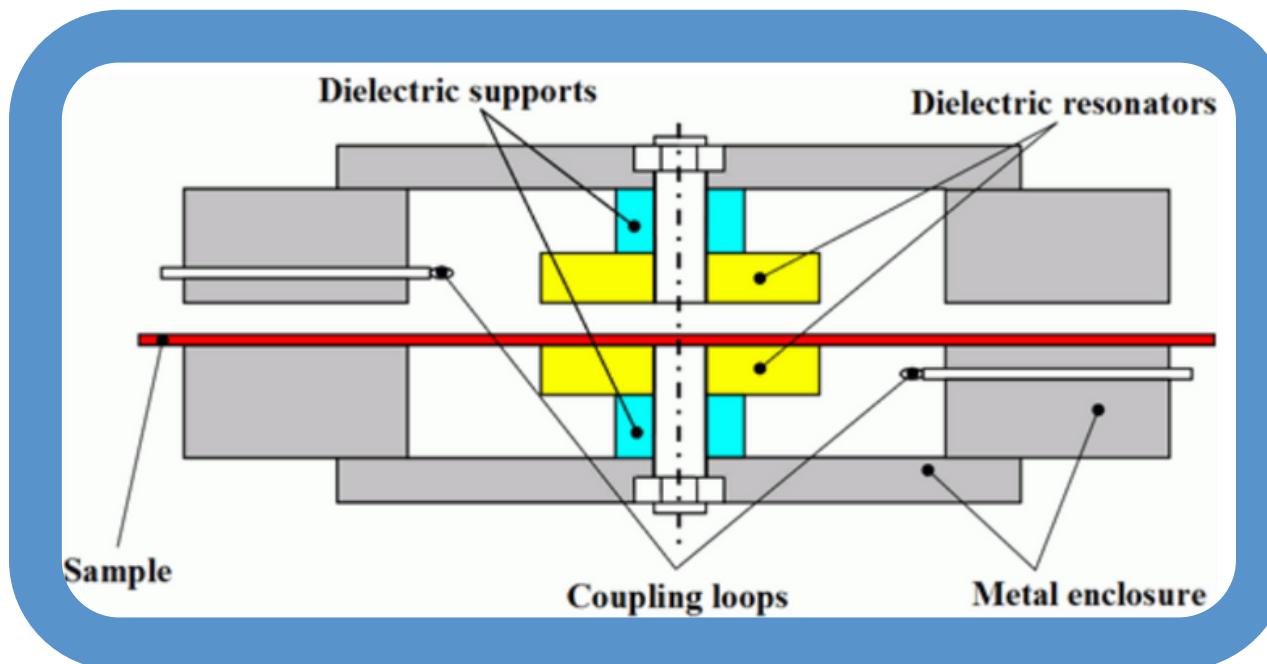
$$\hat{\epsilon} = \epsilon' - i\epsilon''$$

Complex permittivity

Distributions of electric  
and magnetic fields

## Methods for measuring the complex dielectric function

- Parallel Plate
- Coaxial Probe
- Transmission Line
- Free Space
- Resonators



**Single** Post Dielectric  
Resonator

The most common methods for measuring the complex dielectric (and possibly magnetic) function. [6][8]



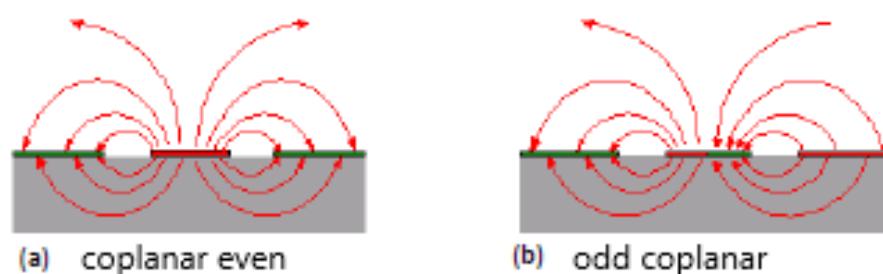
## Description of quasi-TEM waveguides

Distribution of transverse components  
of electric **E** and magnetic **H** fields

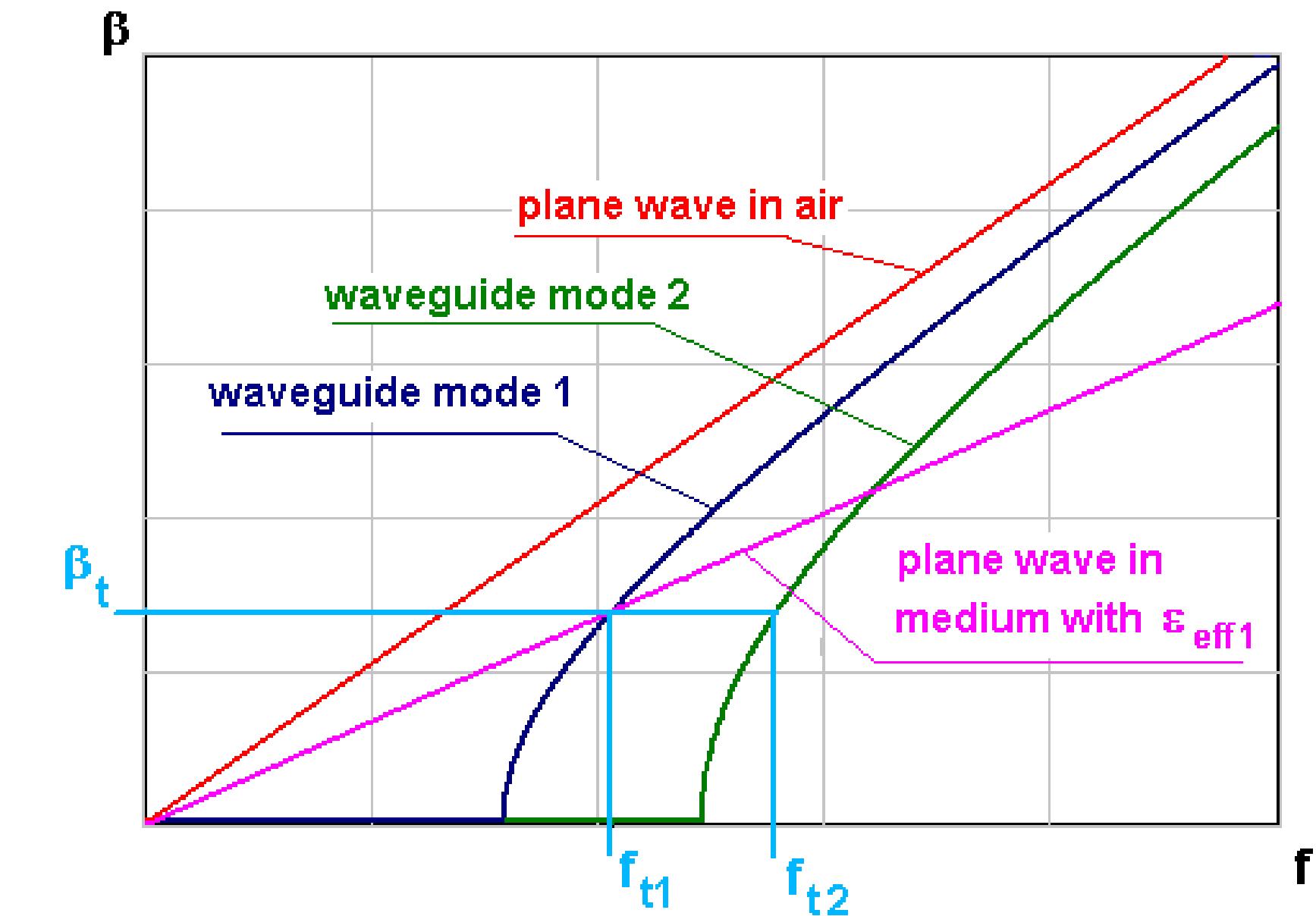
$$\tilde{E}_t(x, y, z) = (c^+ \cdot e^{-\gamma z} + c^- \cdot e^{\gamma z}) \cdot \tilde{e}_t$$

$$\tilde{H}_t(x, y, z) = (c^+ \cdot e^{-\gamma z} - c^- \cdot e^{\gamma z}) \cdot \tilde{h}_t$$

complex propagation  
constant



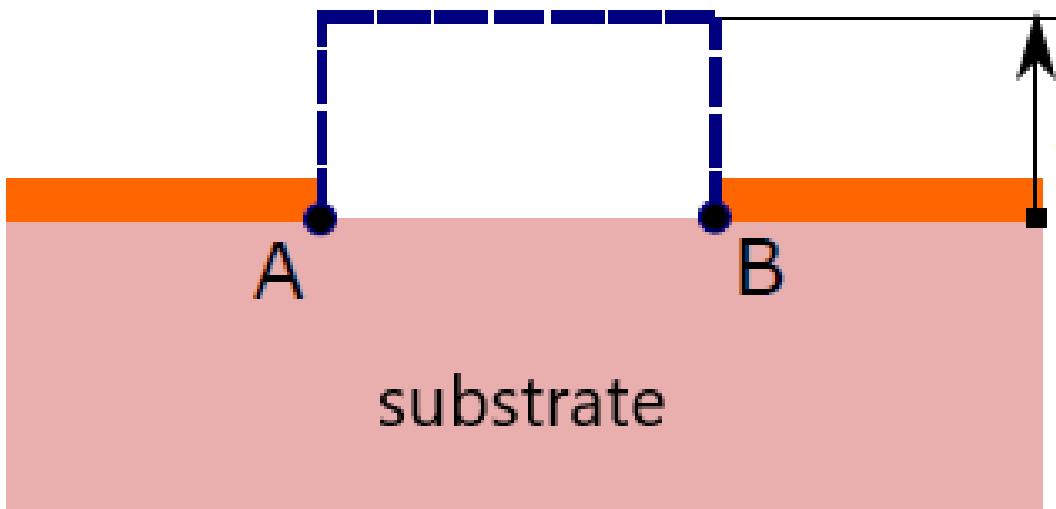
$$\epsilon_{\text{eff}} = \left( \frac{\beta}{k_0} \right)^2$$



First insight template generation

## Description of quasi-TEM waveguides

integration contour



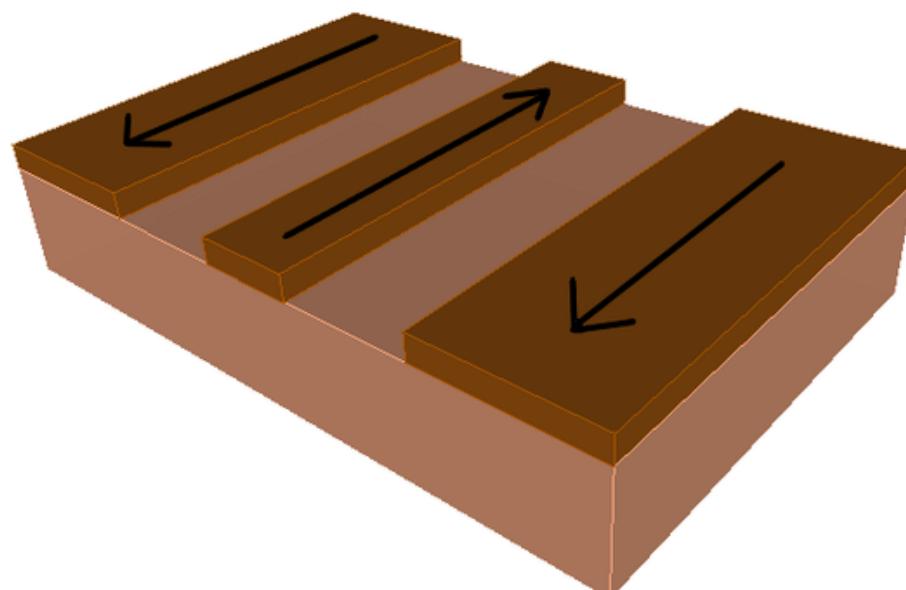
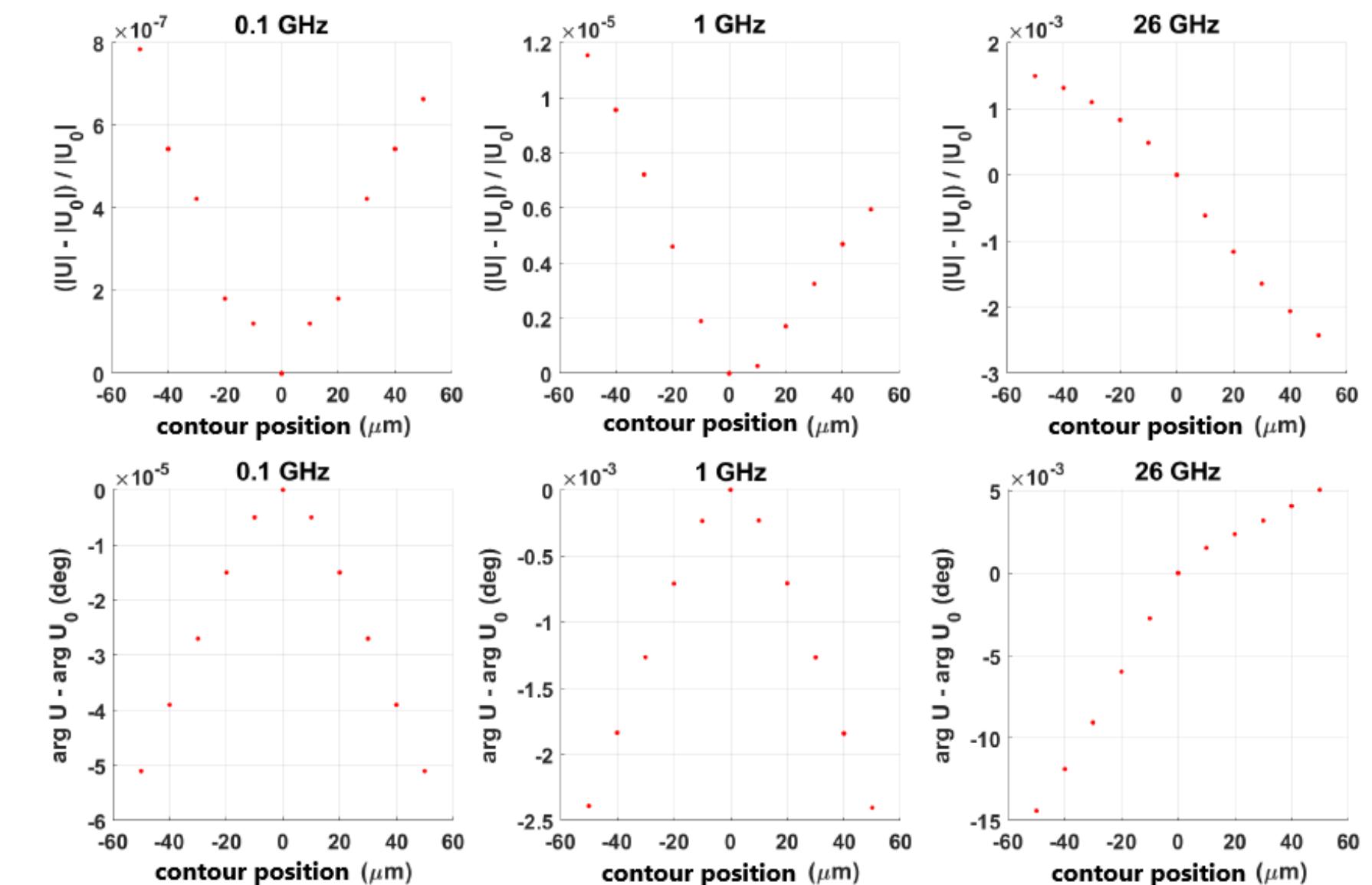
$$v(z) = - \int_A^B \tilde{E}_t(z) \cdot dl$$

$$v_0 = - \int_A^B \tilde{e}_t \cdot dl$$

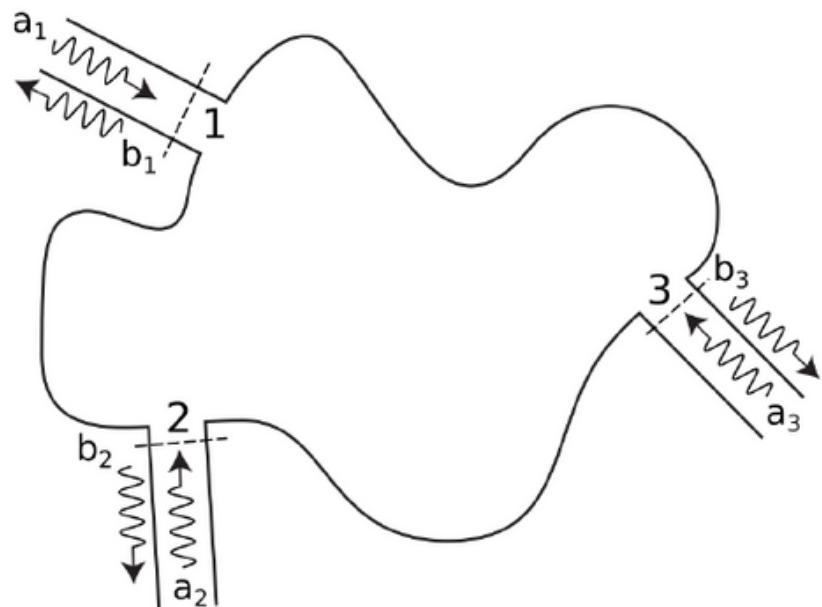
$$v(z) = (c^+ \cdot e^{-\gamma z} + c^- \cdot e^{\gamma z})v_0 \quad i(z) = (c^+ \cdot e^{-\gamma z} - c^- \cdot e^{\gamma z})i_0$$

$$Z_0 = \frac{v_0}{i_0}$$

characteristic impedance [6]



## Scattering matrix



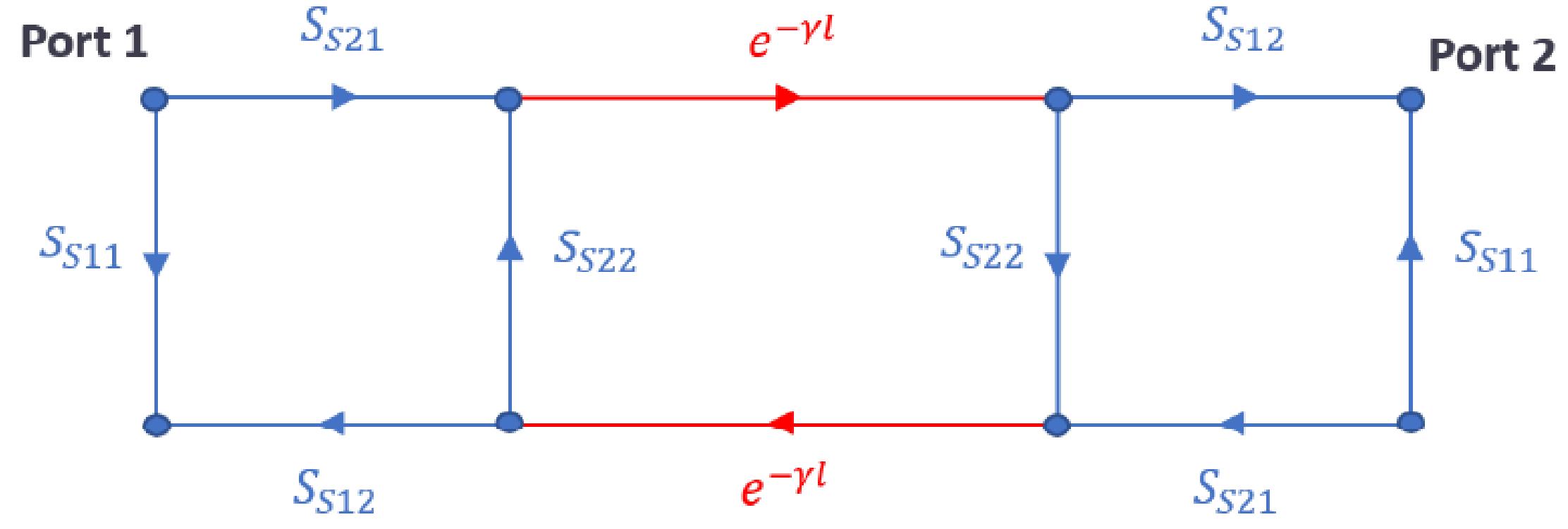
$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{|\Re(Z_i)|}}, \quad b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{|\Re(Z_i)|}}$$

$$P_i = \Re(V_i I_i^*) = |a_i|^2 - |b_i|^2$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nn} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

$$Z_i = Z_R = \Re(Z_R)$$

$$a_i = c_i^+ \cdot \frac{v_0}{\sqrt{Z_R}}, \quad b_i = c_i^- \cdot \frac{v_0}{\sqrt{Z_R}}$$



Impedance step  $Z_R - Z_0$    CPW segment of length  $l$    Impedance step  $Z_0 - Z_R$

Blue arrows point to the equations for  $S_{11}$  and  $S_{21}$ :

$$S_{11} = S_{S11} \frac{1 - e^{-2\gamma l}}{1 - S_{S11}^2 e^{-2\gamma l}} = S_{22}$$

$$S_{21} = (1 - S_{S11}^2) \frac{e^{-\gamma l}}{1 - S_{S11}^2 e^{-2\gamma l}} = S_{12}$$

Red arrows point to the equations for  $S_{S11}$  and  $S_{S21}$ :

$$S_{S11} = \frac{Z_0 - Z_R}{Z_0 + Z_R}, \quad S_{S12} = \frac{2Z_R}{Z_0 + Z_R}$$

$$S_{S21} = \frac{2Z_0}{Z_0 + Z_R}, \quad S_{S12} = \frac{Z_R - Z_0}{Z_0 + Z_R}$$

Red box contains the formula for  $Z_0$ :

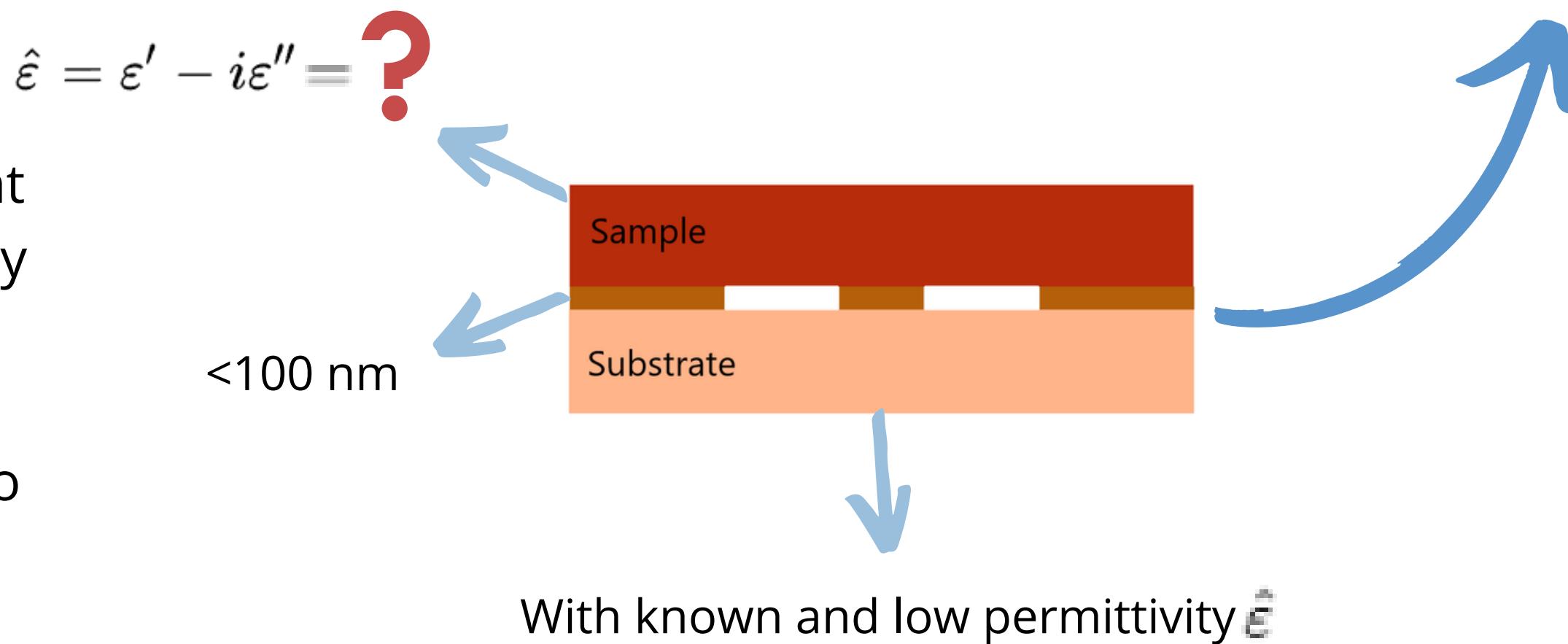
$$Z_0 = Z_R \sqrt{\frac{S_{21}^2 - (1 + S_{11})^2}{S_{21}^2 - (1 - S_{11})^2}}$$

## Proposal for measuring thin films

**CPW** waveguide, as a quasi-TEM guide, is adequate to study interaction of composites with plane electromagnetic waves.

Significant advantages:

- simplicity of the measurement
  - it is enough to place directly on the surface of the conductive strips
- low sensitivity of the results to geometric dimensions of the sample



# Which EM parameters meaningfully describe thin film interaction with GHz waves?

$$R_s = \frac{(d_0 \sigma_0)^{-1}}{d} = (d \sigma)^{-1}$$

parameters of the original metal      dielectric surrogate

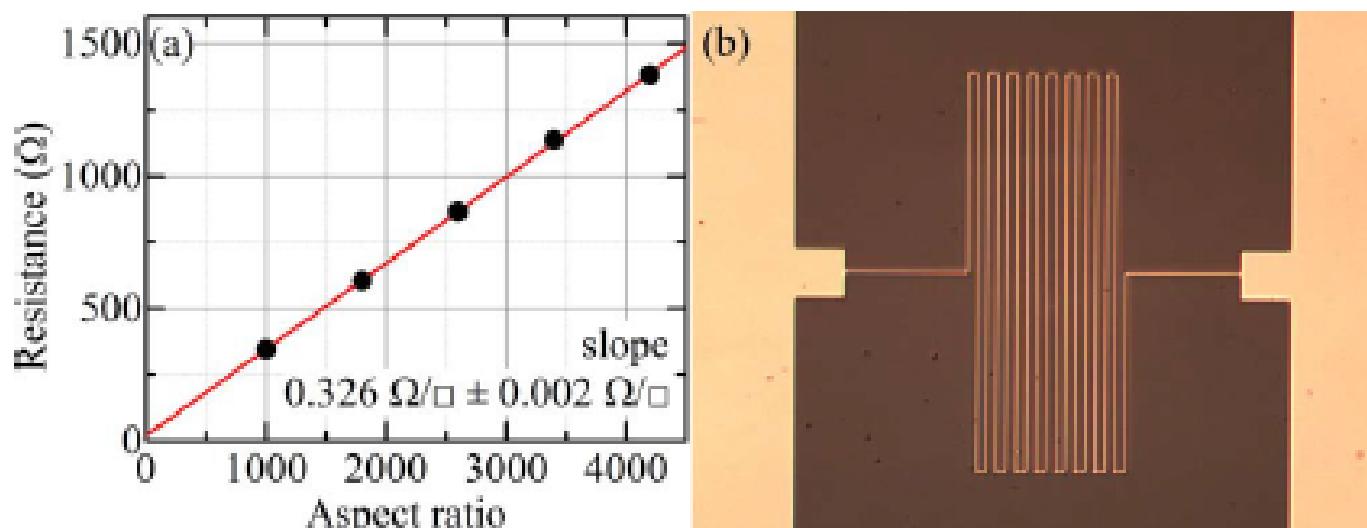
Scaling:

- Anisotropic
- Impedance-conserving

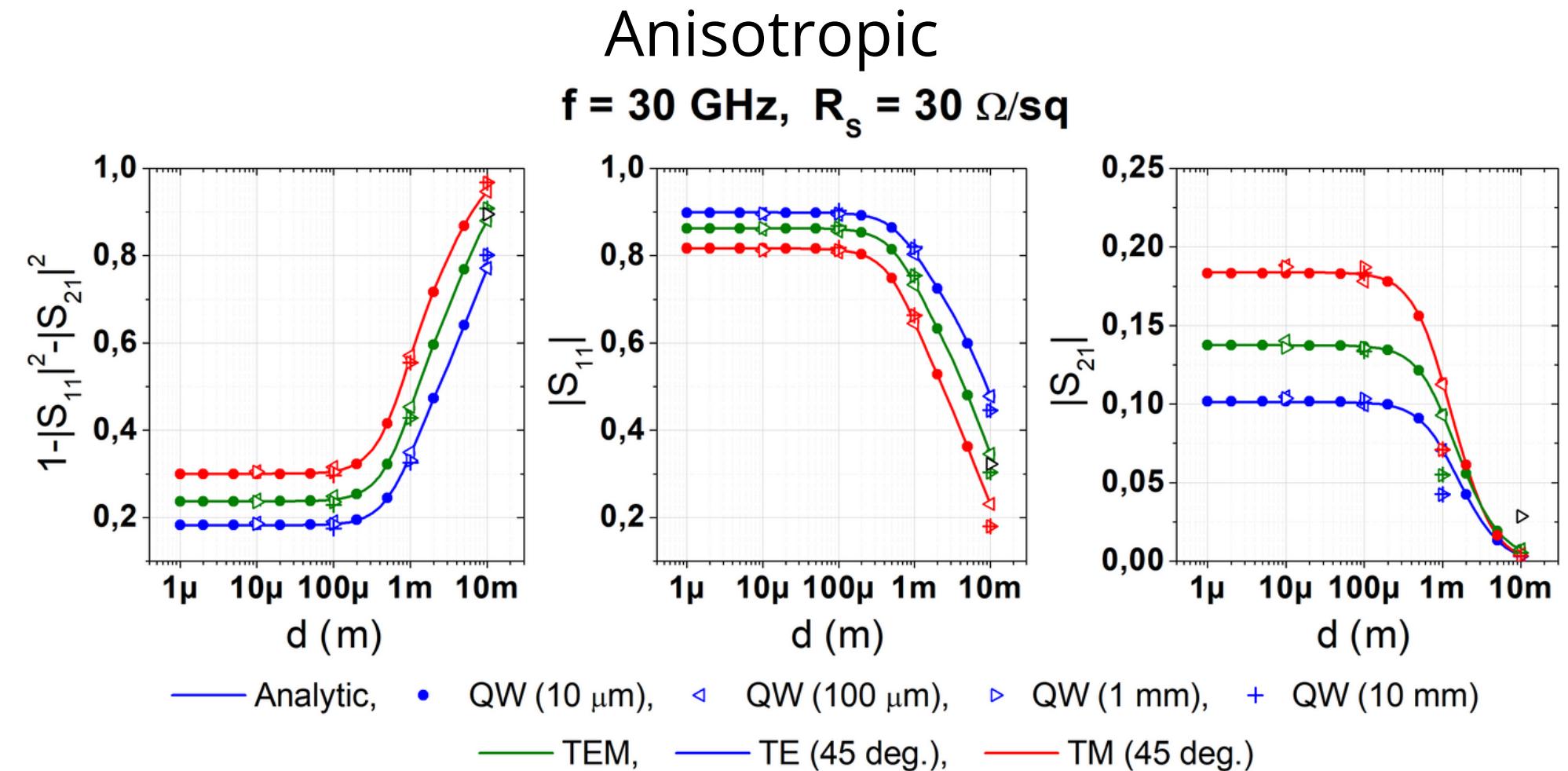
Specifically, sub-cellular models in conformal FDTD are recommended for the modelling of minimally thin surrogate sheets with acceptable computer effort.

METALLIZATION SHEET RESISTANCE/CONDUCTIVITY

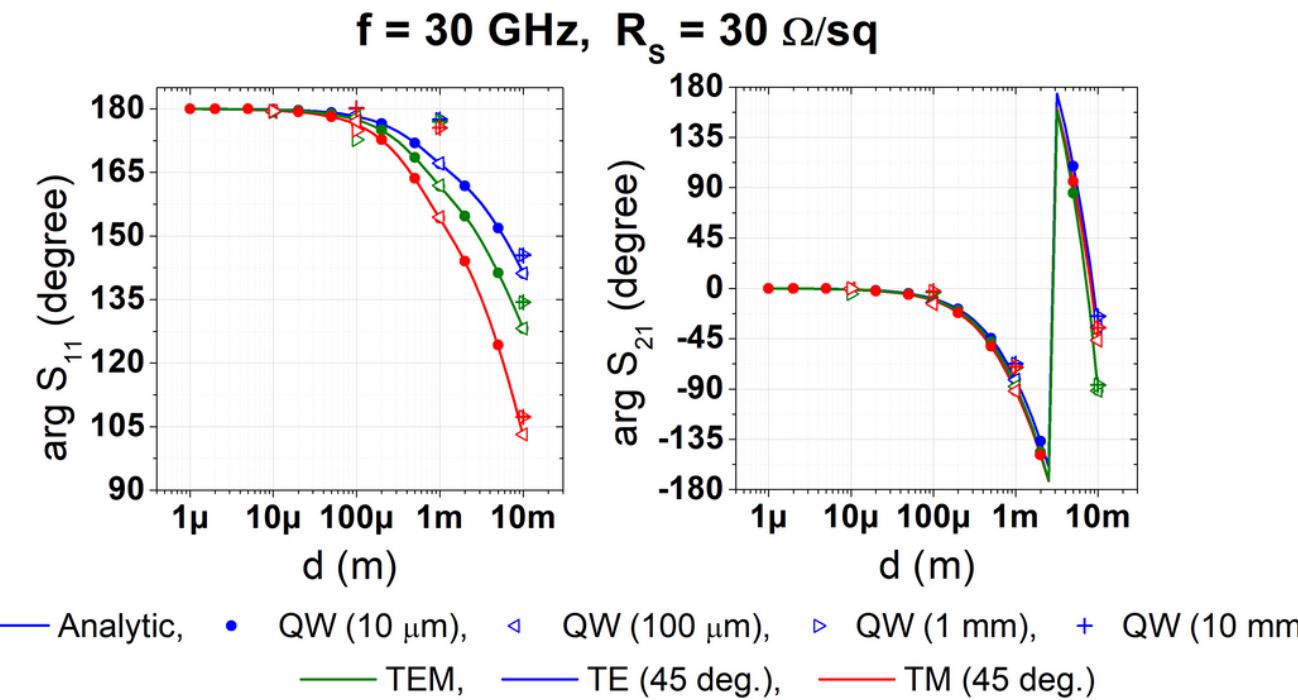
Metallization	$R_{S,DC}$ ( $\Omega/\square$ )	$R_{S,RF}$ ( $\Omega/\square$ )	$\sigma_{DC}$ ( $10^7$ $\text{S}/\text{m}$ )	$\sigma_{RF}$ ( $10^7$ $\text{S}/\text{m}$ )
5 nm Cr + 10 nm Au	6.86	8.69	0.972	0.767
5 nm Cr + 20 nm Au	2.53	2.91	1.58	1.37
5 nm Cr + 30 nm Au	1.44	1.73	1.98	1.65
5 nm Cr + 50 nm Au	0.710	0.814	2.56	2.23
5 nm Cr + 70 nm Au	0.487	0.528	2.74	2.53
5 nm Cr + 100 nm Au	0.326	0.346	2.92	2.75



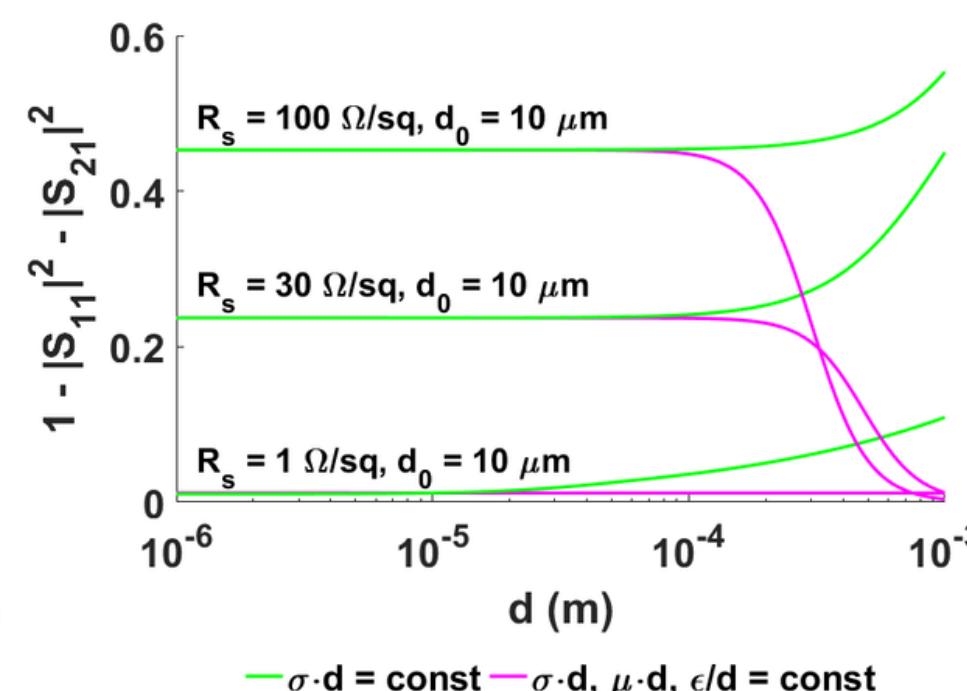
(a) Resistance of the thin metallic strip versus the aspect ratio (length divided by width); slope equals the sheet resistance.  
 (b) Picture of one of the metallic structures used in the Transfer Length Method for determining the metallization dc sheet resistance for 5-nm Cr + 100-nm Au metallization. [4]



### Anisotropic



### Impedance-conserving



Comparisons of the original metal layers and their scaled surrogates:

- Absorption is retained for surrogates thicker up to  $d=0.02\lambda$ , but nearly doubles when  $d=0.1\lambda$
- Accuracy is improved by anisotropic scaling, but only for TM waves and thick surrogates ( $d=10\text{mm}$ , black mark in the figures above).
- Magnitudes as well as phases of reflection ( $\sim 180^\circ$ ) and transmission ( $\sim 0^\circ$ ) are retained for  $d < 0.02\lambda$ ; at  $d=0.1\lambda$  phase difference ca.  $10^\circ$  between the polarisations is caused, which may visibly modify field pattern above and below the surrogate
- Impedance-conserving models allow a broader range of scaling for low-resistance sheets as in aeronautics, but fail for high-resistance susceptors:

# CPW modelling with the FDTD method

## Introduction

### Faraday and Ampere

$$\frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}$$

$$\frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_x$$

$$\frac{\partial D_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - J_y$$

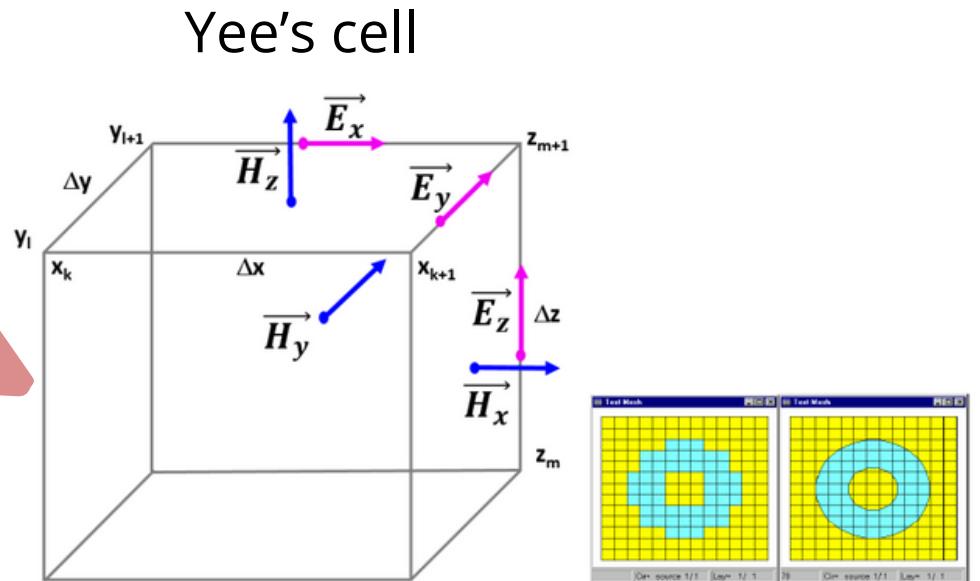
$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z$$

discretization  
of equations

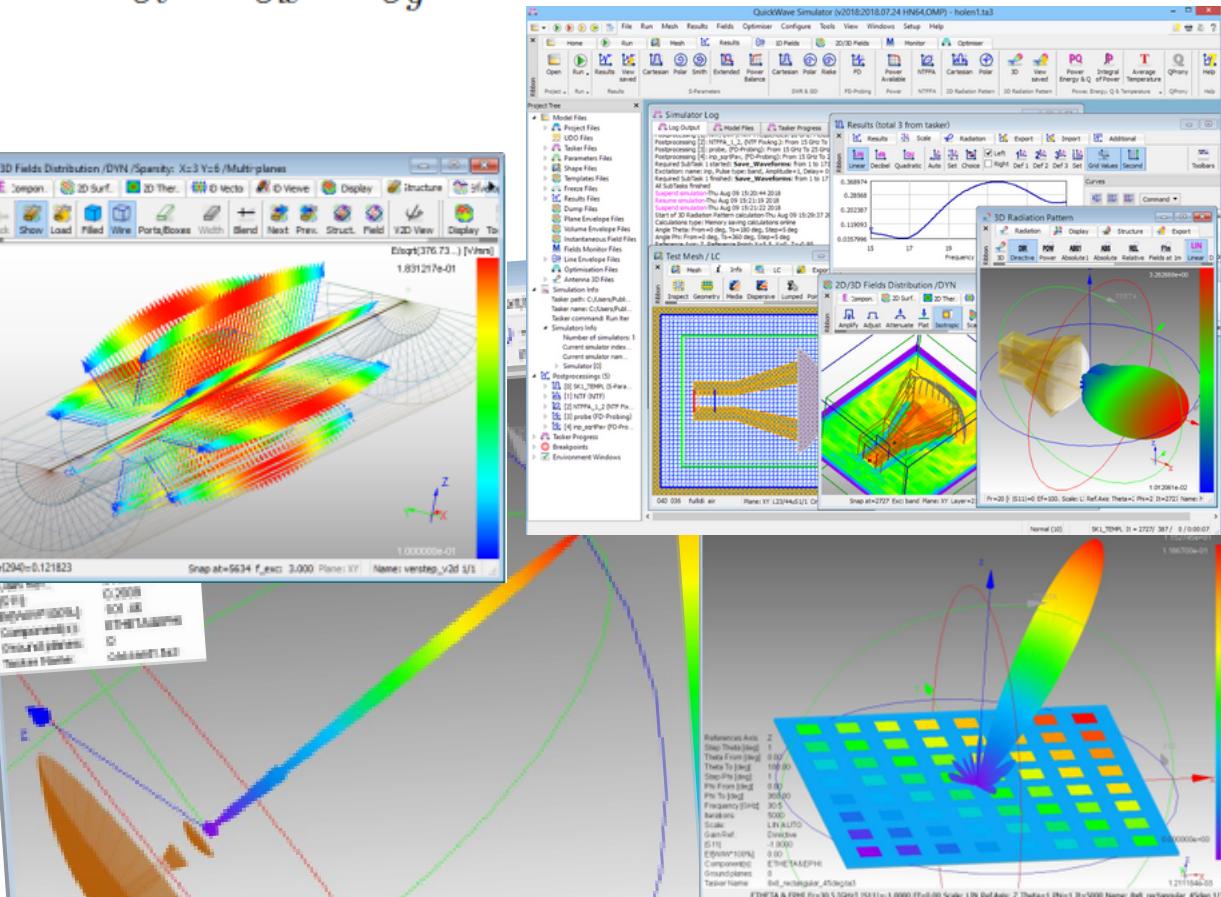
$$\frac{B_{x; i, j+1/2, k+1/2}^{n+1/2} - B_{x; i, j+1/2, k+1/2}^{n-1/2}}{\Delta t} = \frac{E_{y; i, j+1/2, k+1}^n - E_{y; i, j+1/2, k}^n}{\Delta z} - \frac{E_{z; i, j+1, k+1/2}^n - E_{z; i, j, k+1/2}^n}{\Delta y}$$

$$\frac{D_{x; i+1/2, j, k}^n - D_{x; i+1/2, j, k}^{n-1}}{\Delta t} = \frac{H_{z; i+1/2, j+1/2, k}^{n-1/2} - H_{z; i+1/2, j-1/2, k}^{n-1/2}}{\Delta y} - \frac{H_{y; i+1/2, j, k+1/2}^{n-1/2} - H_{y; i+1/2, j, k-1/2}^{n-1/2}}{\Delta z} - J_{x; i+1/2, j, k}^{n-1/2}$$

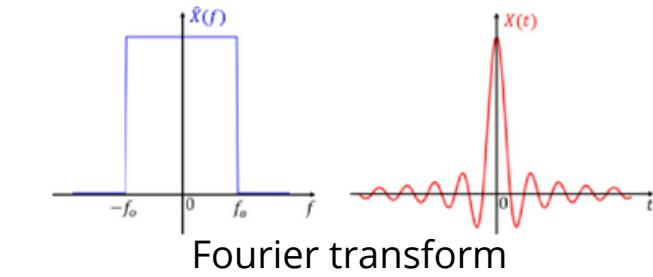
meshing



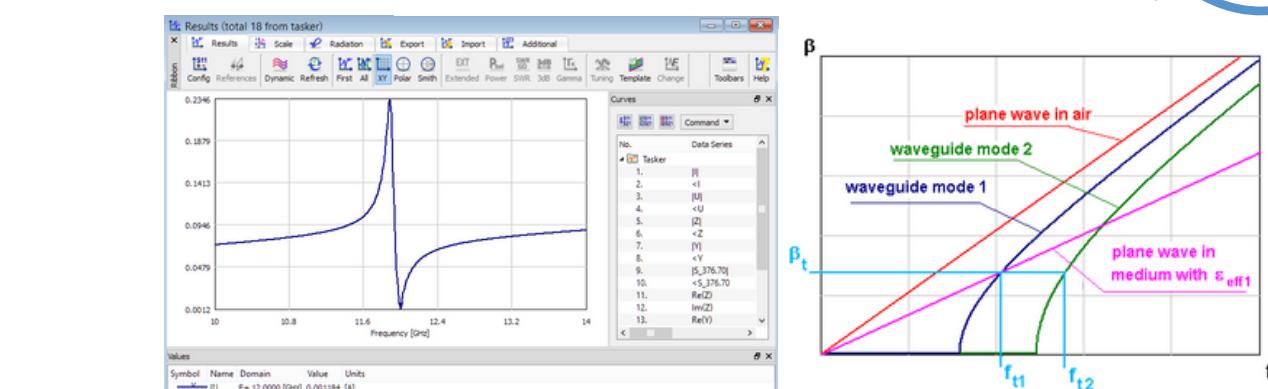
Conformal FDTD engine



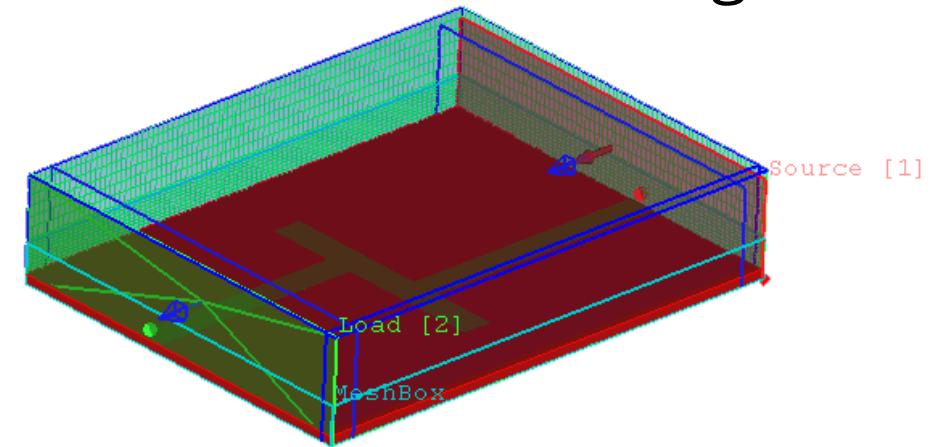
simulation



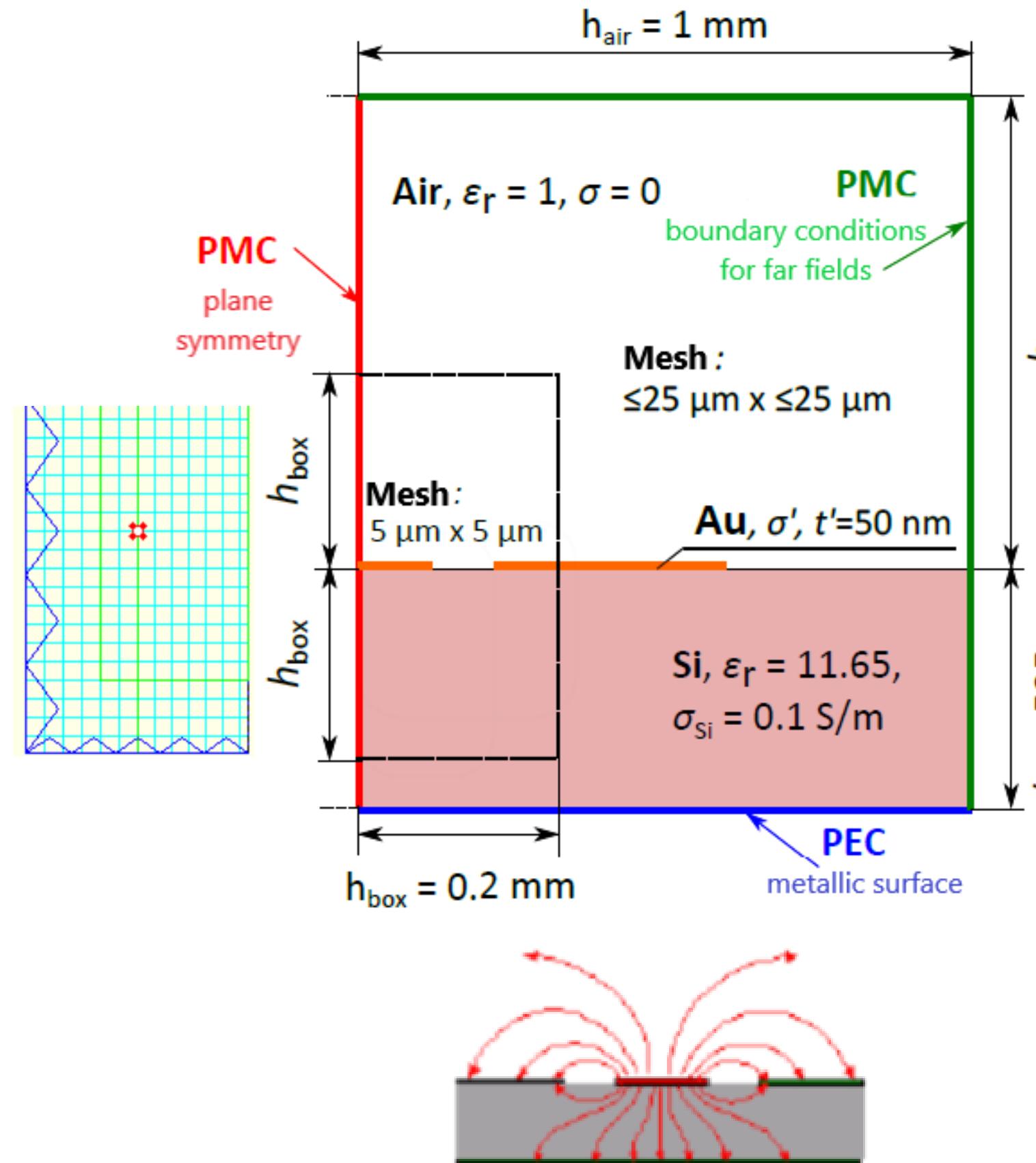
$$v\Delta t \leq 1/\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}$$



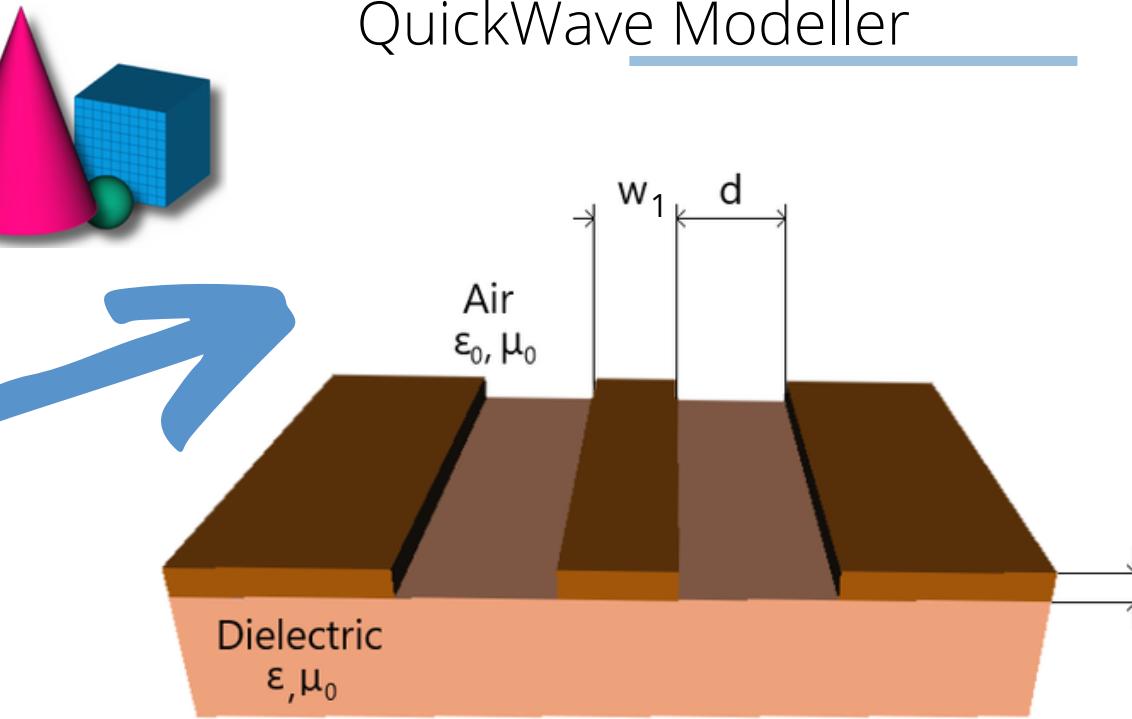
determination of the  
excitation field pattern



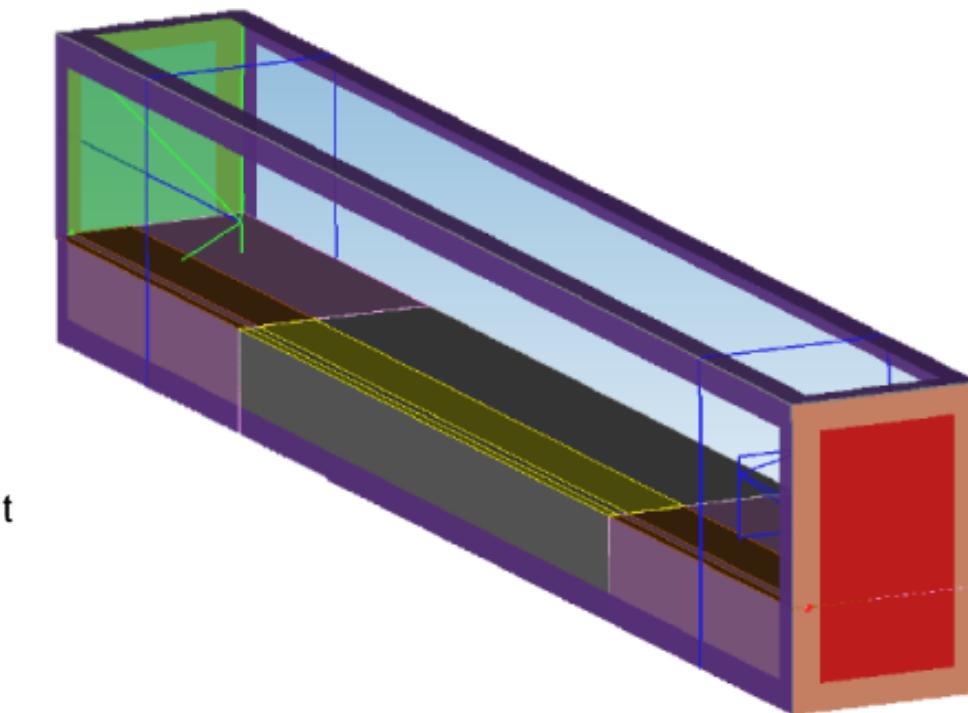
## Modelling CPW in QuickWave



Design in  
QuickWave Modeller



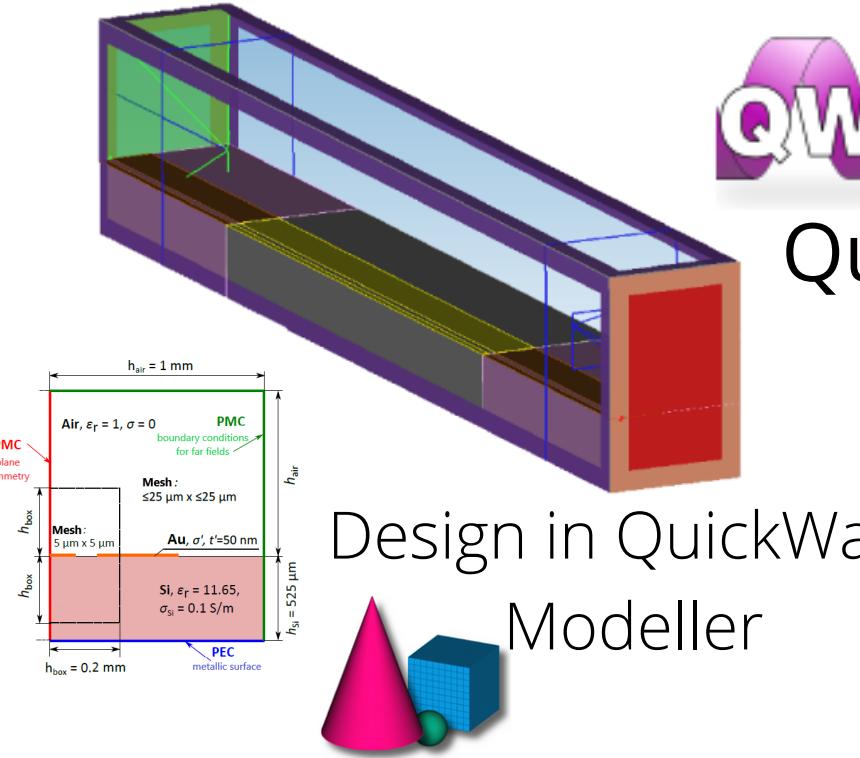
Model with measuring probes



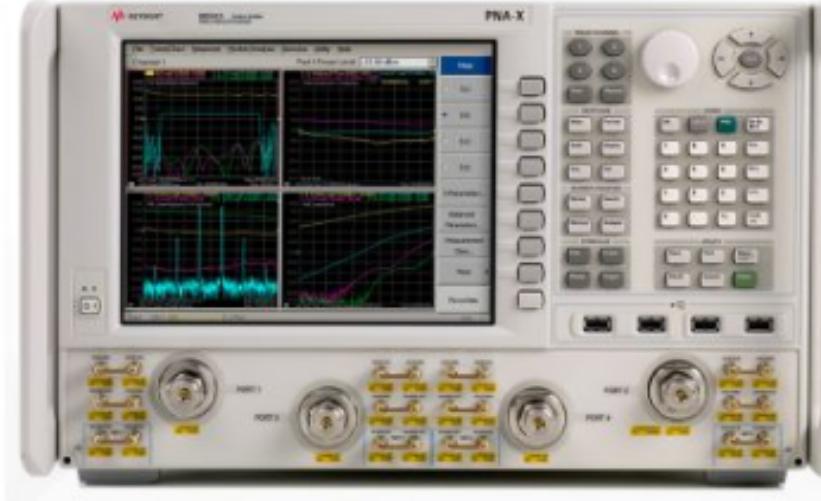
## Assumptions:

- CPW length is equal to **4 mm**, measuring probes length is equal to **2 mm**
- The width of the outer strip is equal to  $W_2 = 100 \mu\text{m} / 300 \mu\text{m}$
- Cell size: **5 μm x 5 μm x 100 μm / 25 μm x 25 μm x 100 μm**
- Excitation field patterns were generated for **f = 10 GHz**
- excitation pulse with a rectangular frequency spectrum from **0 to 26 GHz**, lasting **0.23 ns** (about 23 000 iterations)
- Analysis of half waveguide due to its symmetry
- **PMC** and **PEC** boundary conditions
- Gold and chromium coplanar strips were treated as one with averaged conductivity  $t = t_{\text{Au}} + t_{\text{Cr}}$
- The silicon substrate was assumed to be homogeneous and isotropic
- Maintains the condition  $\sigma \cdot t = \sigma' \cdot t'$

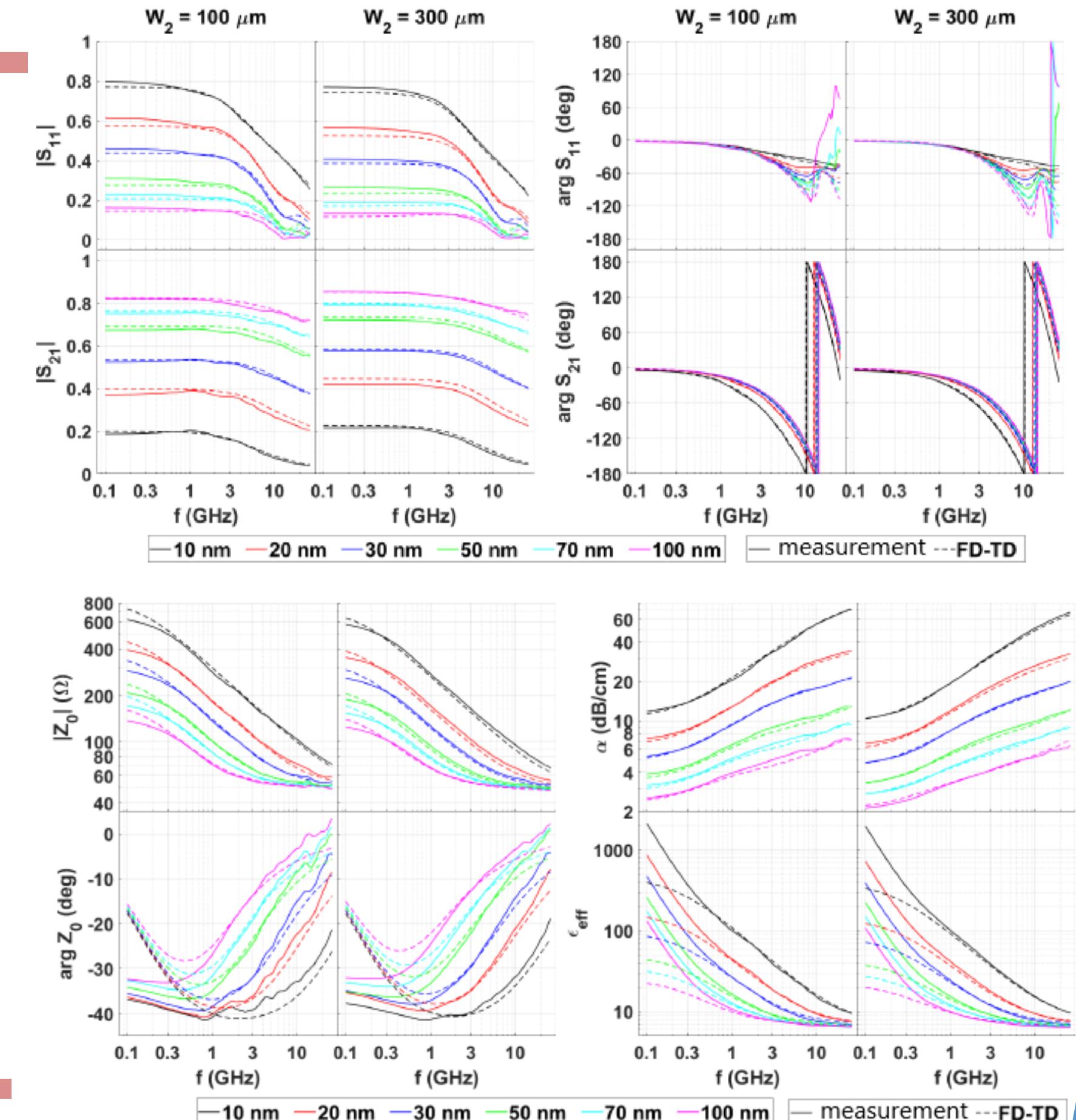
## CPW with measuring probes



## Vector Network Analyzer

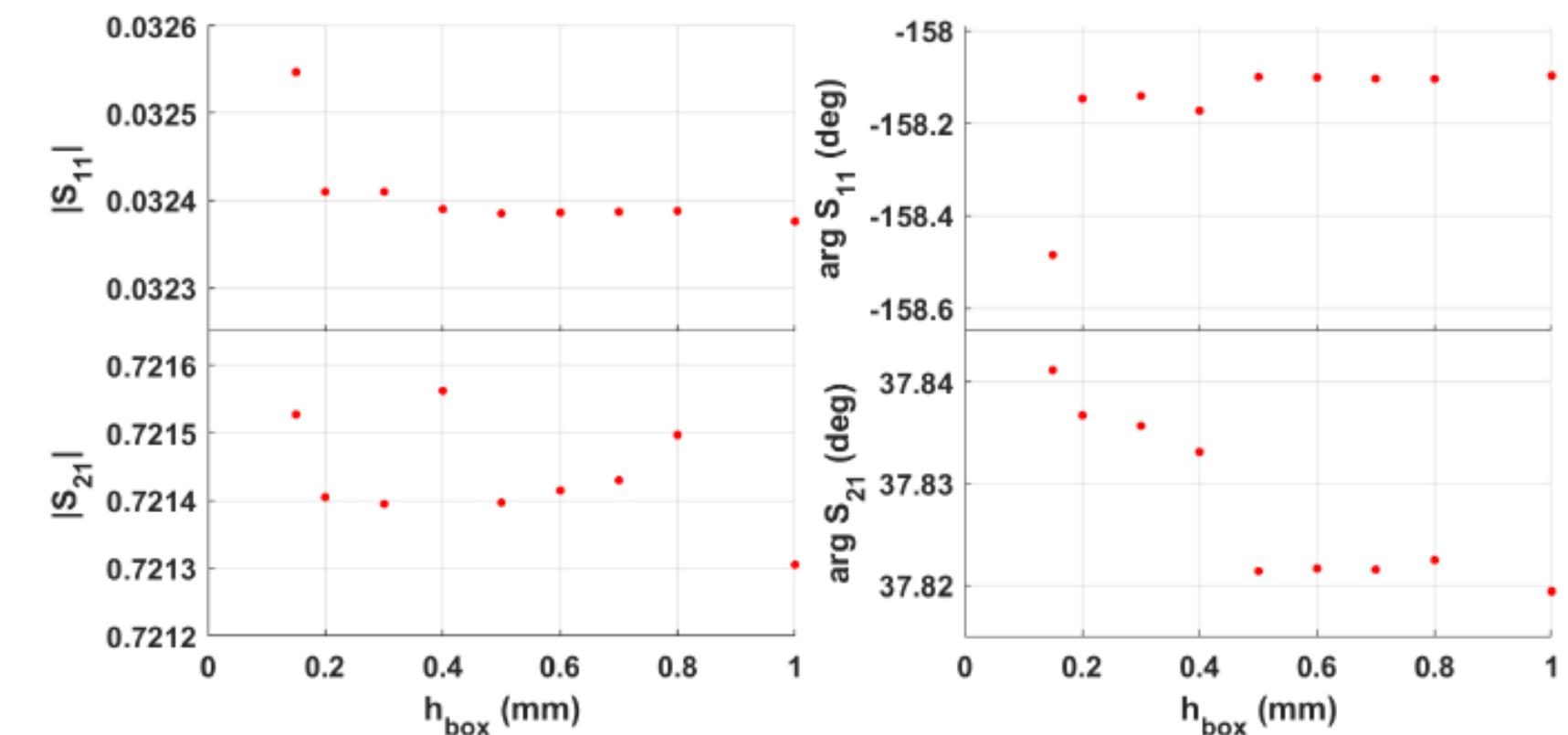
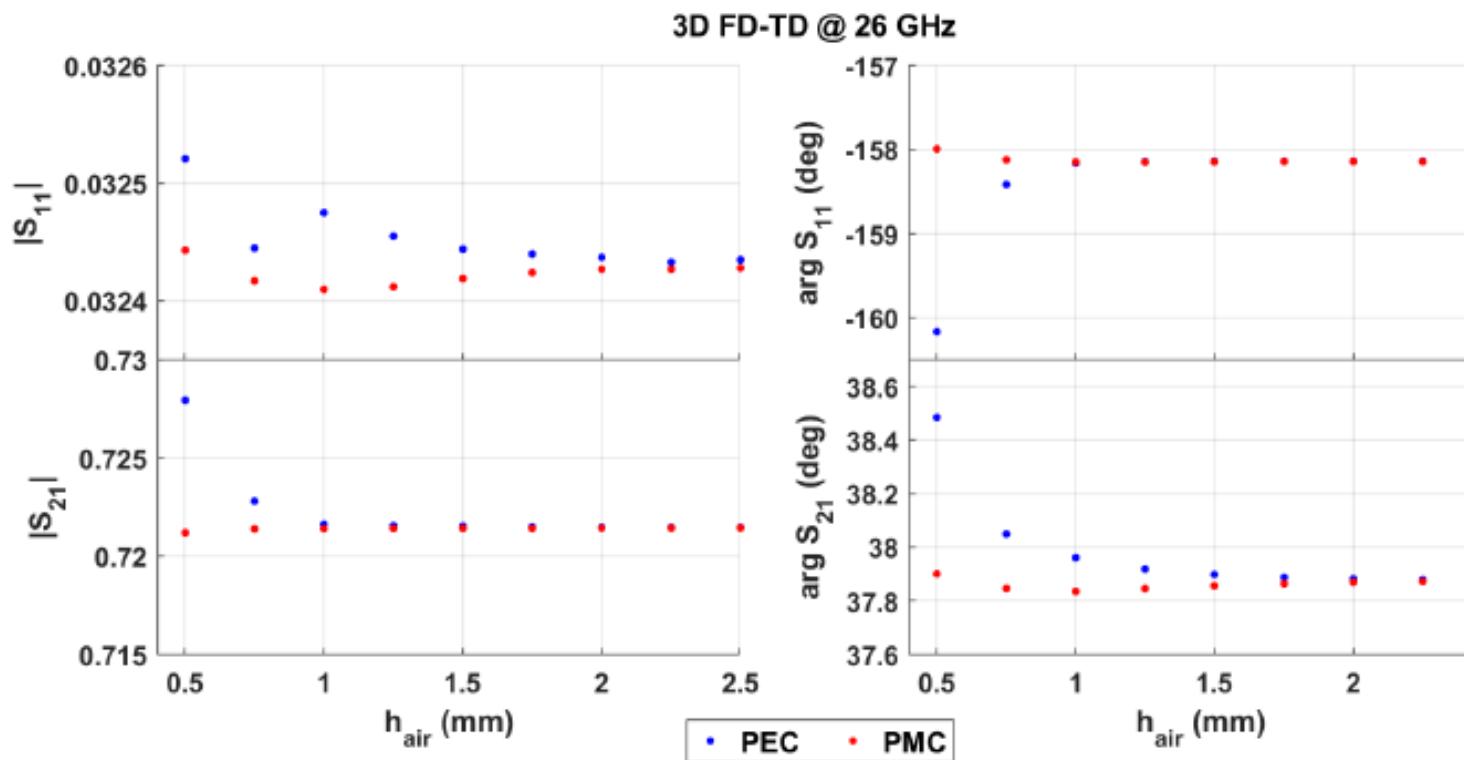
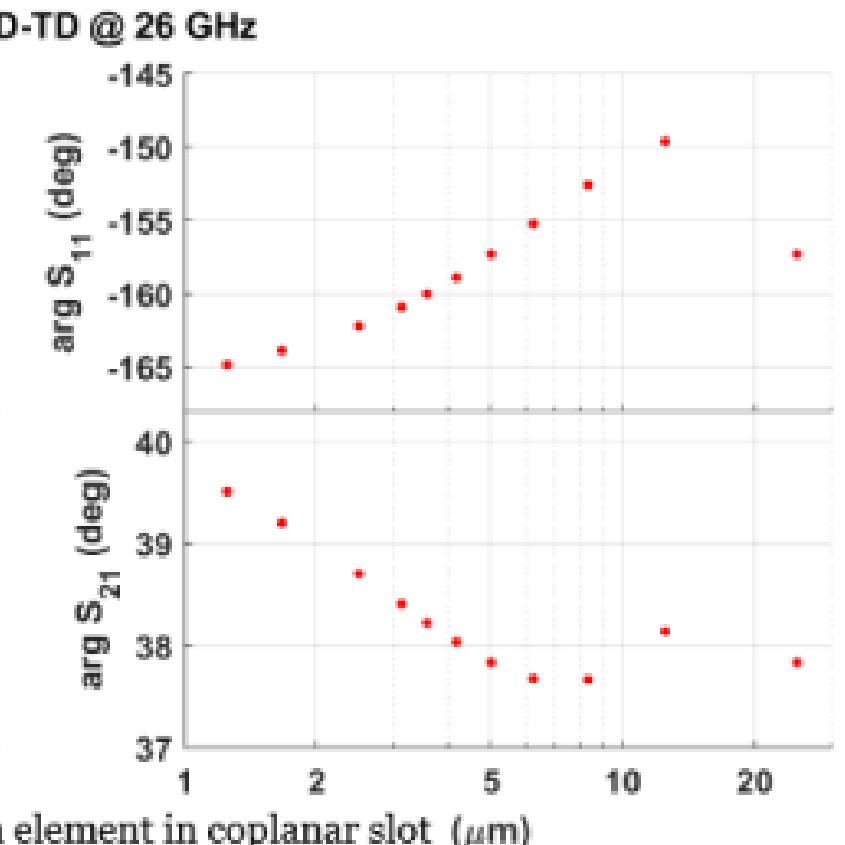
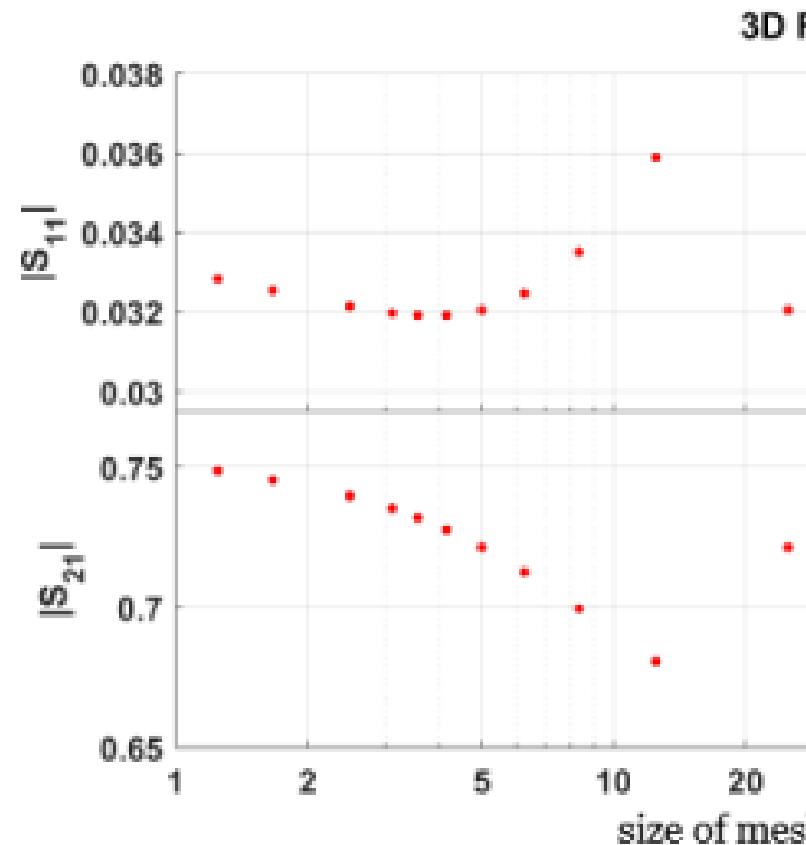
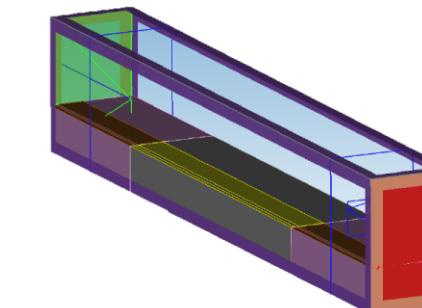
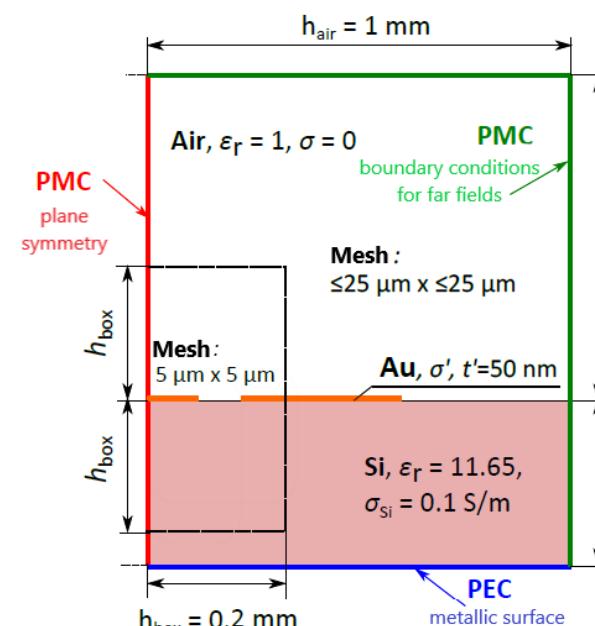
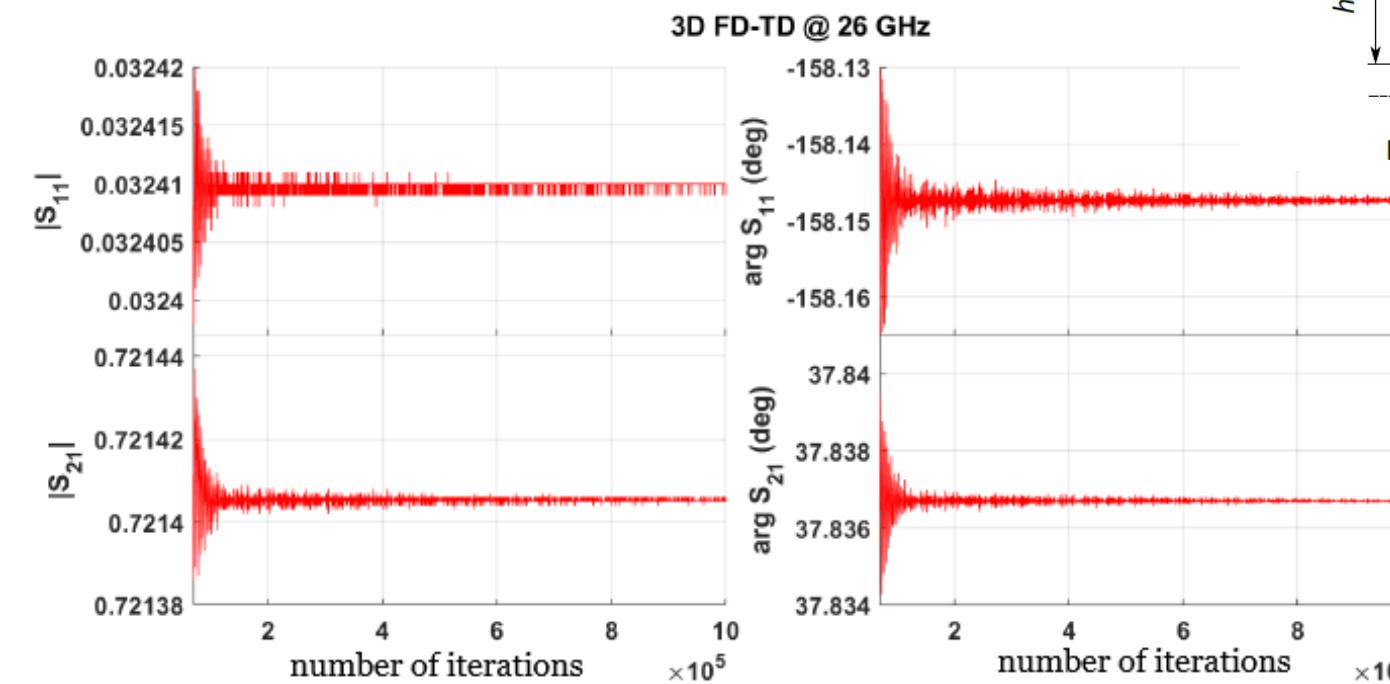


## Agilent N5242A PNA-X [4][7]



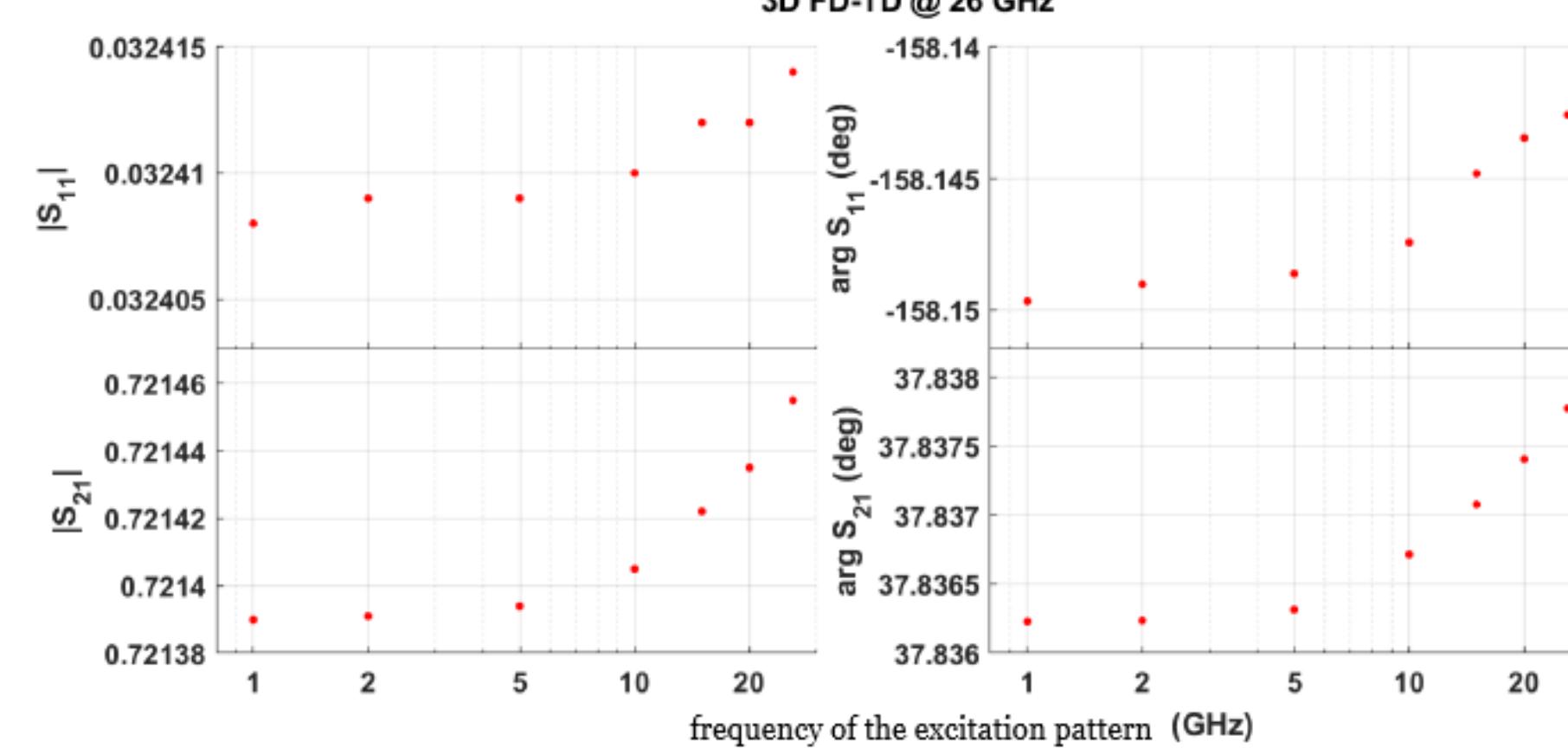
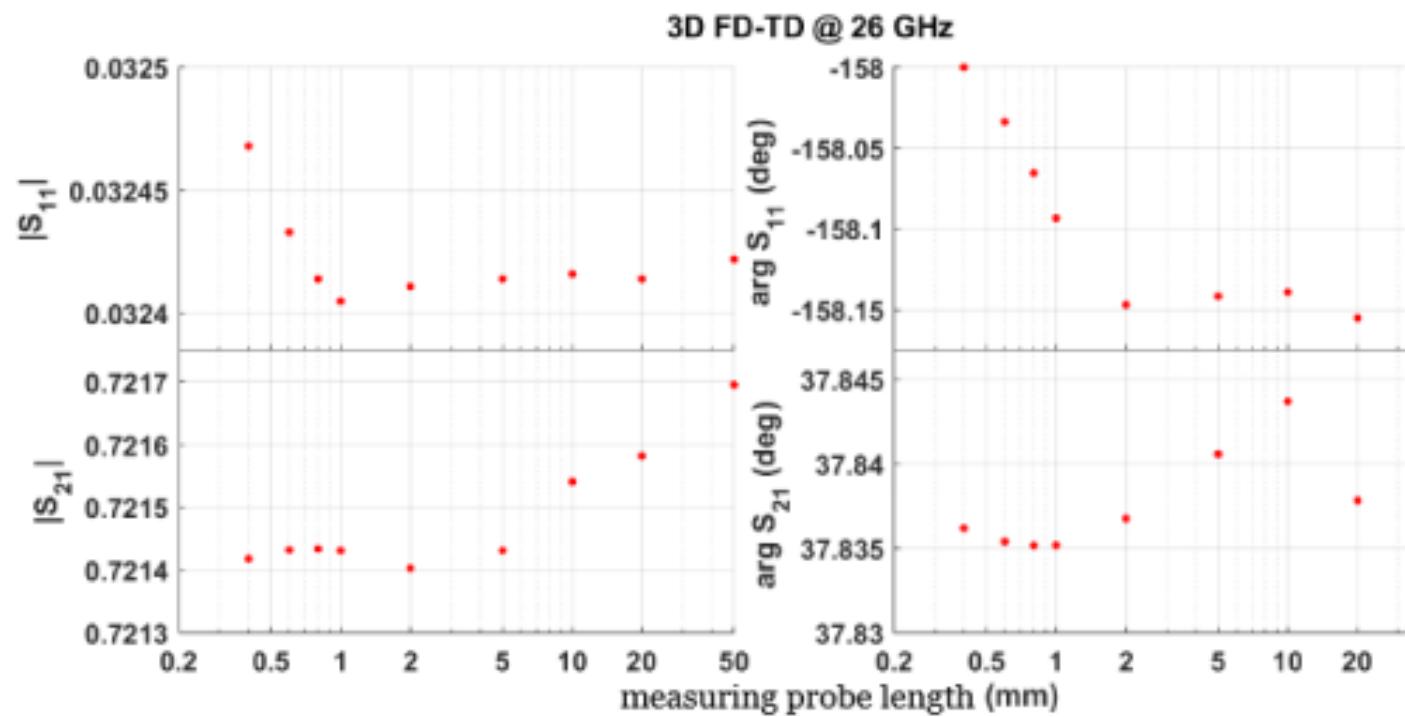
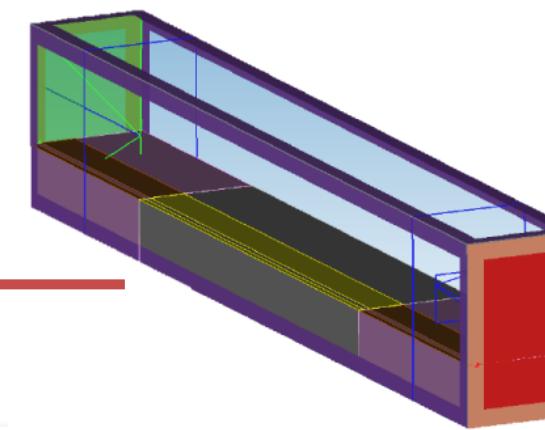
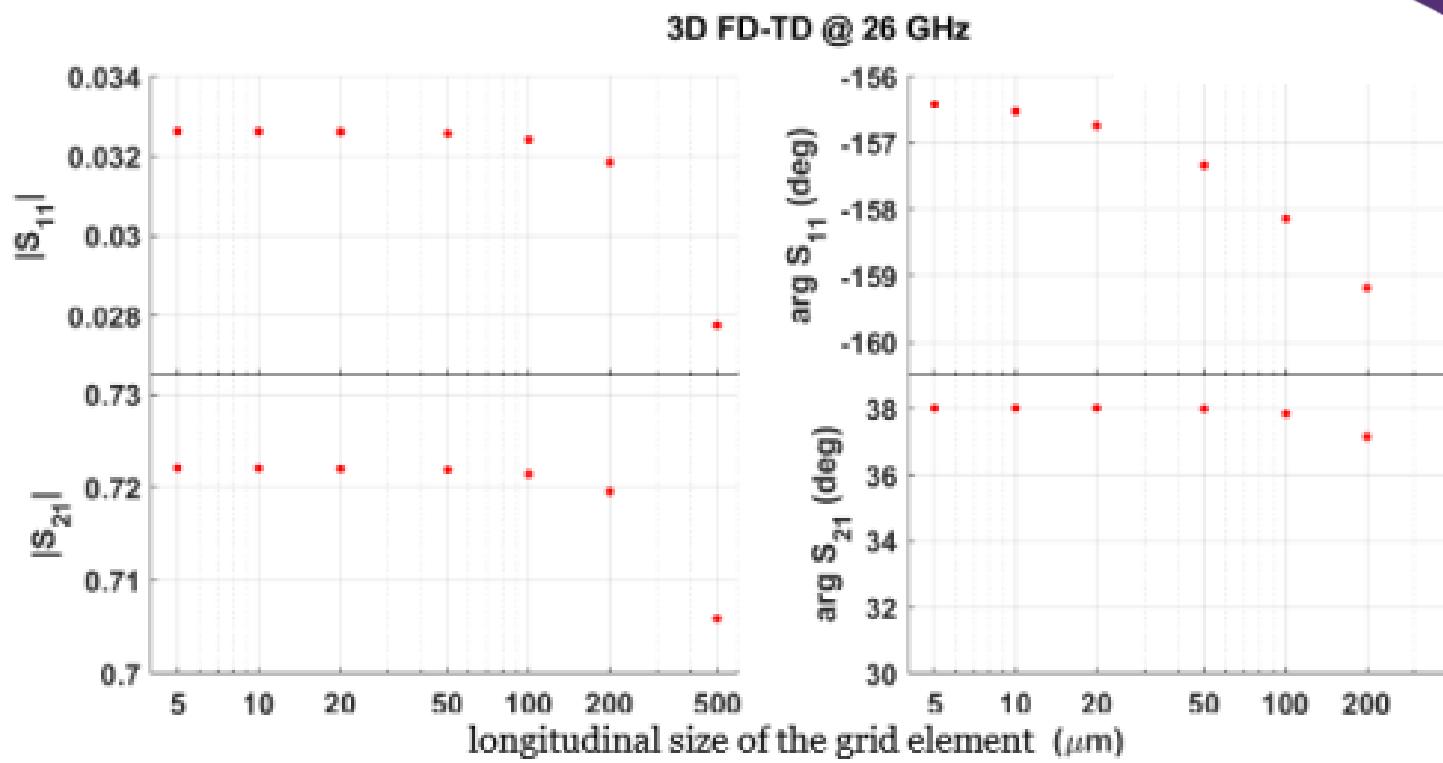
## Convergence tests

### Scattering Matrix



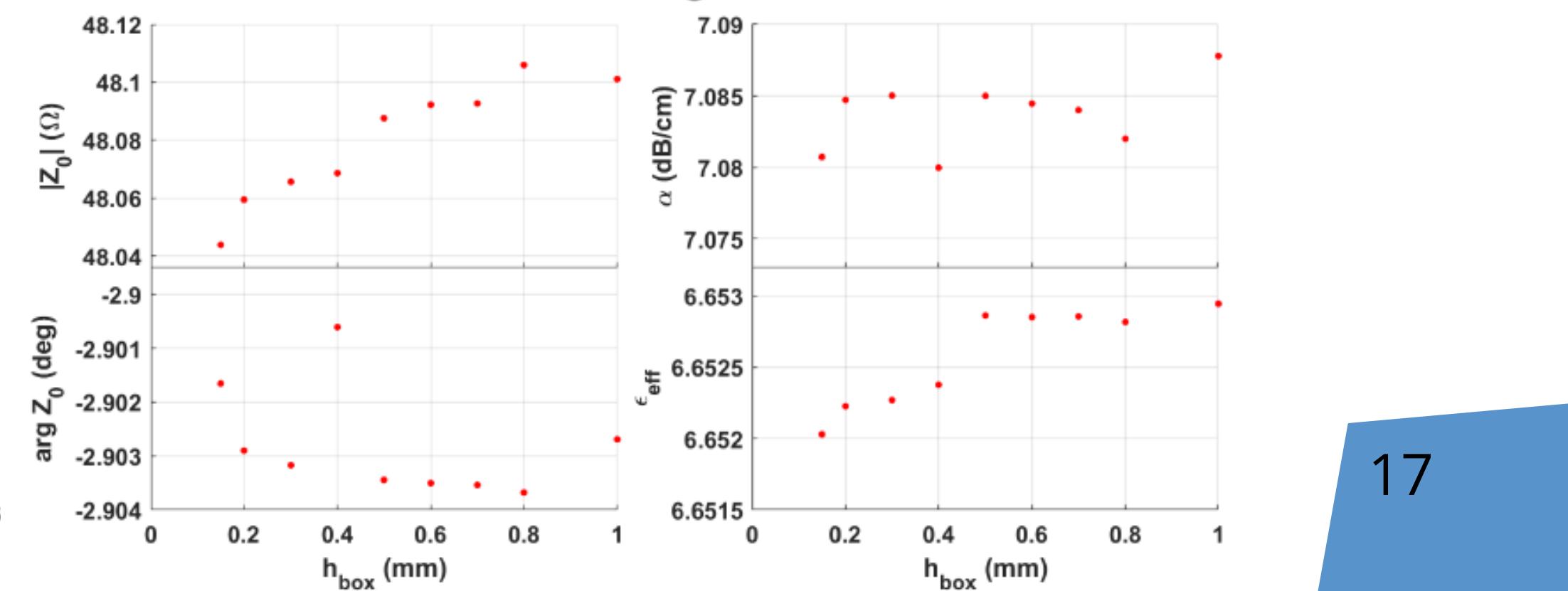
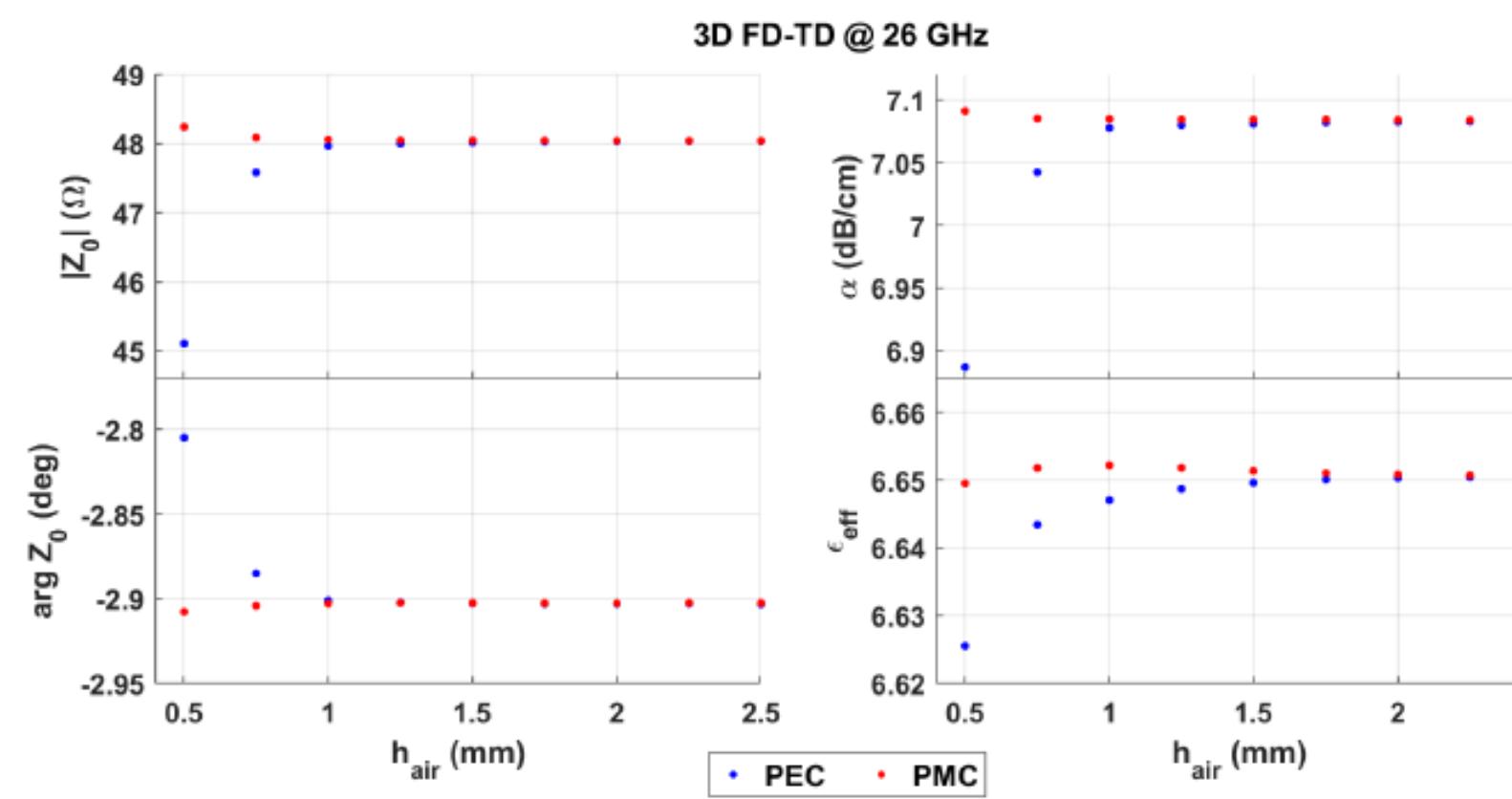
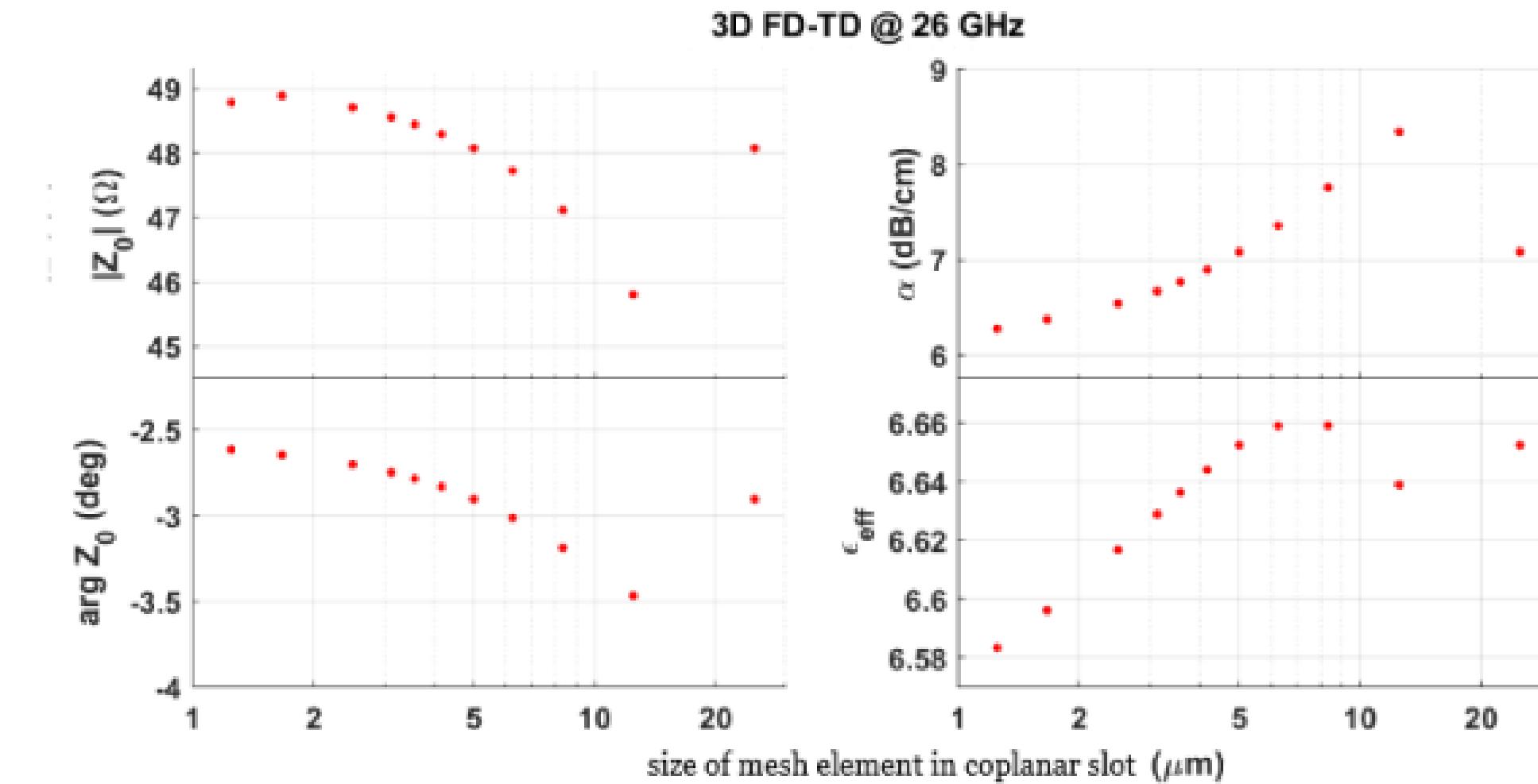
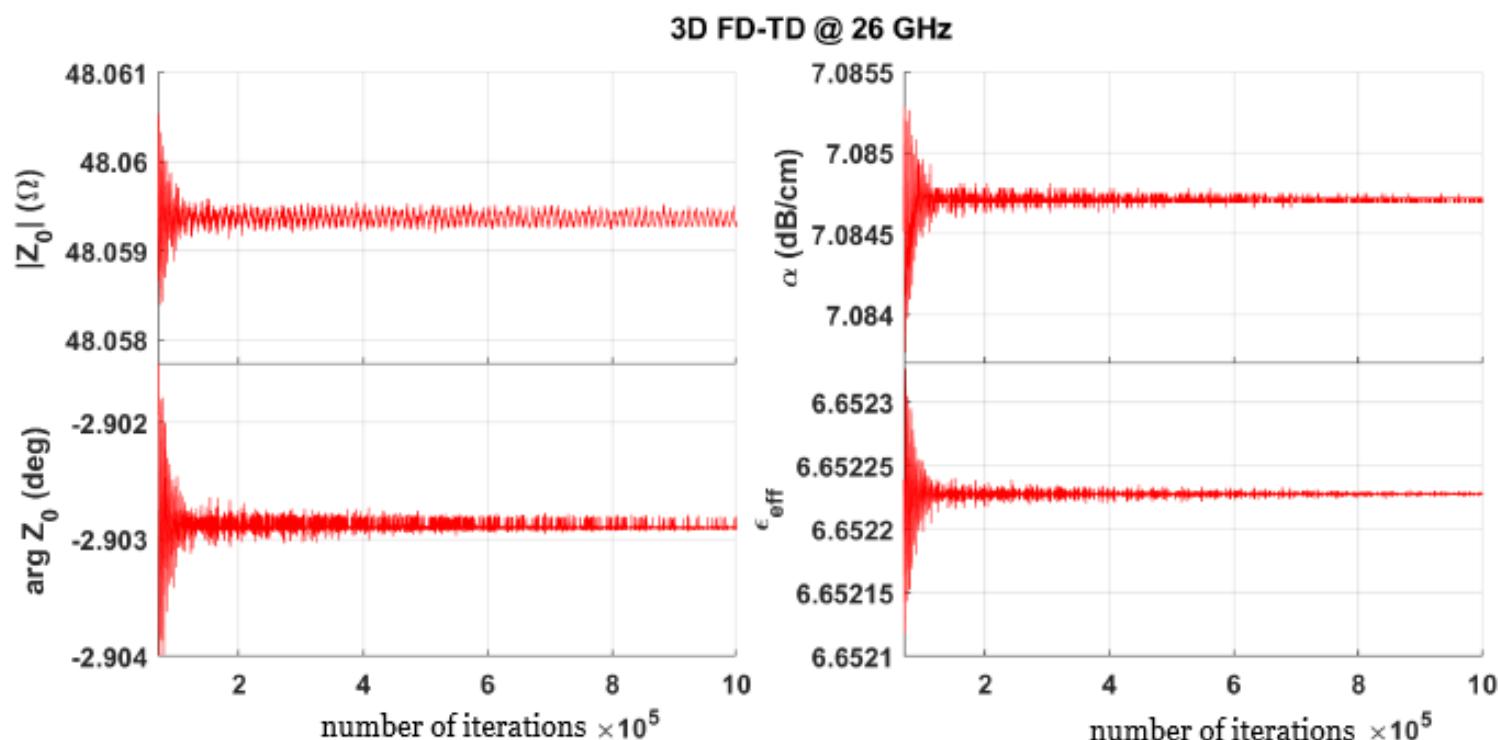
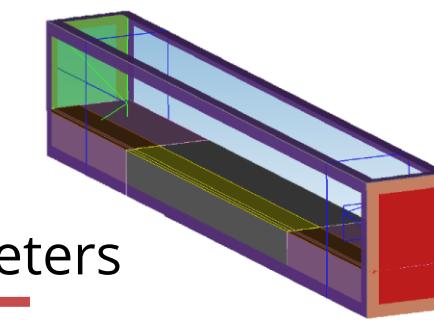
## Convergence tests

## Scattering Matrix



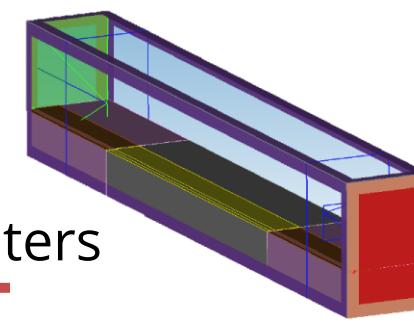
## Convergence tests

## Characteristic parameters

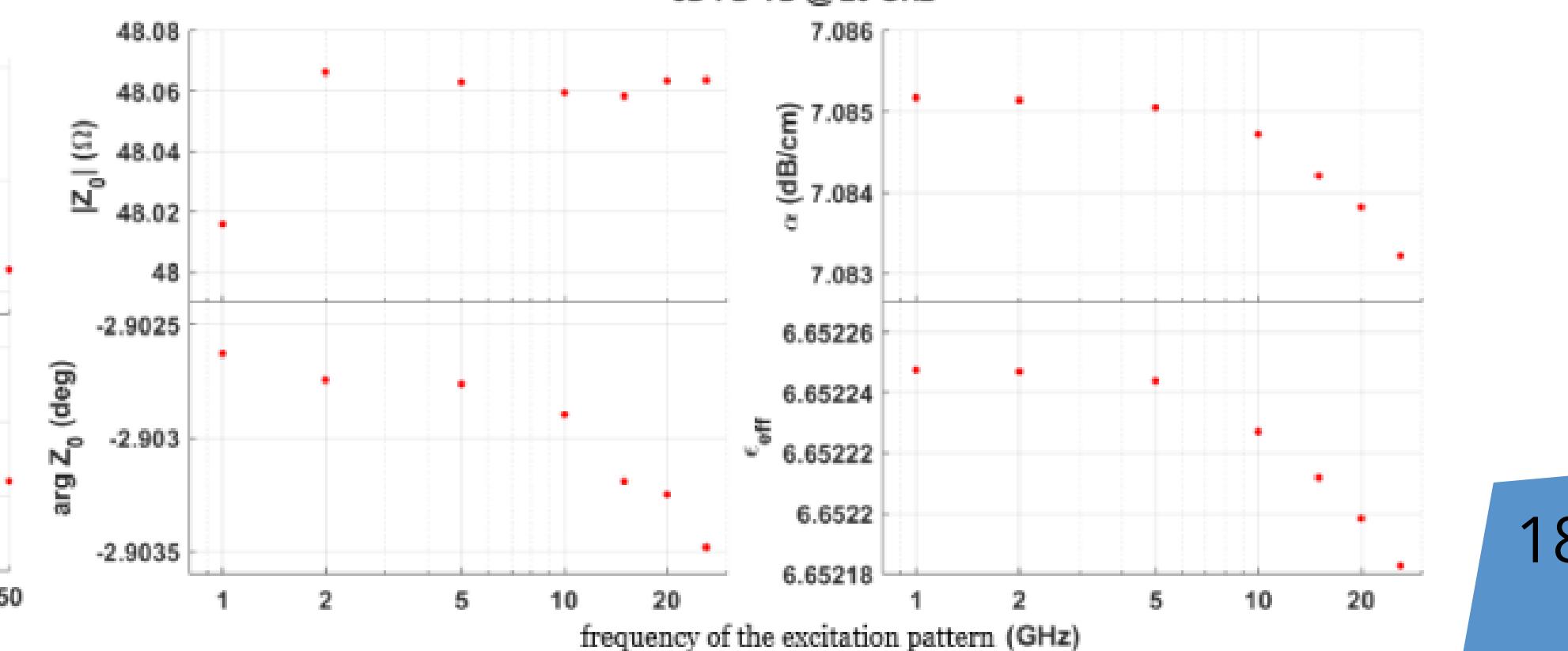
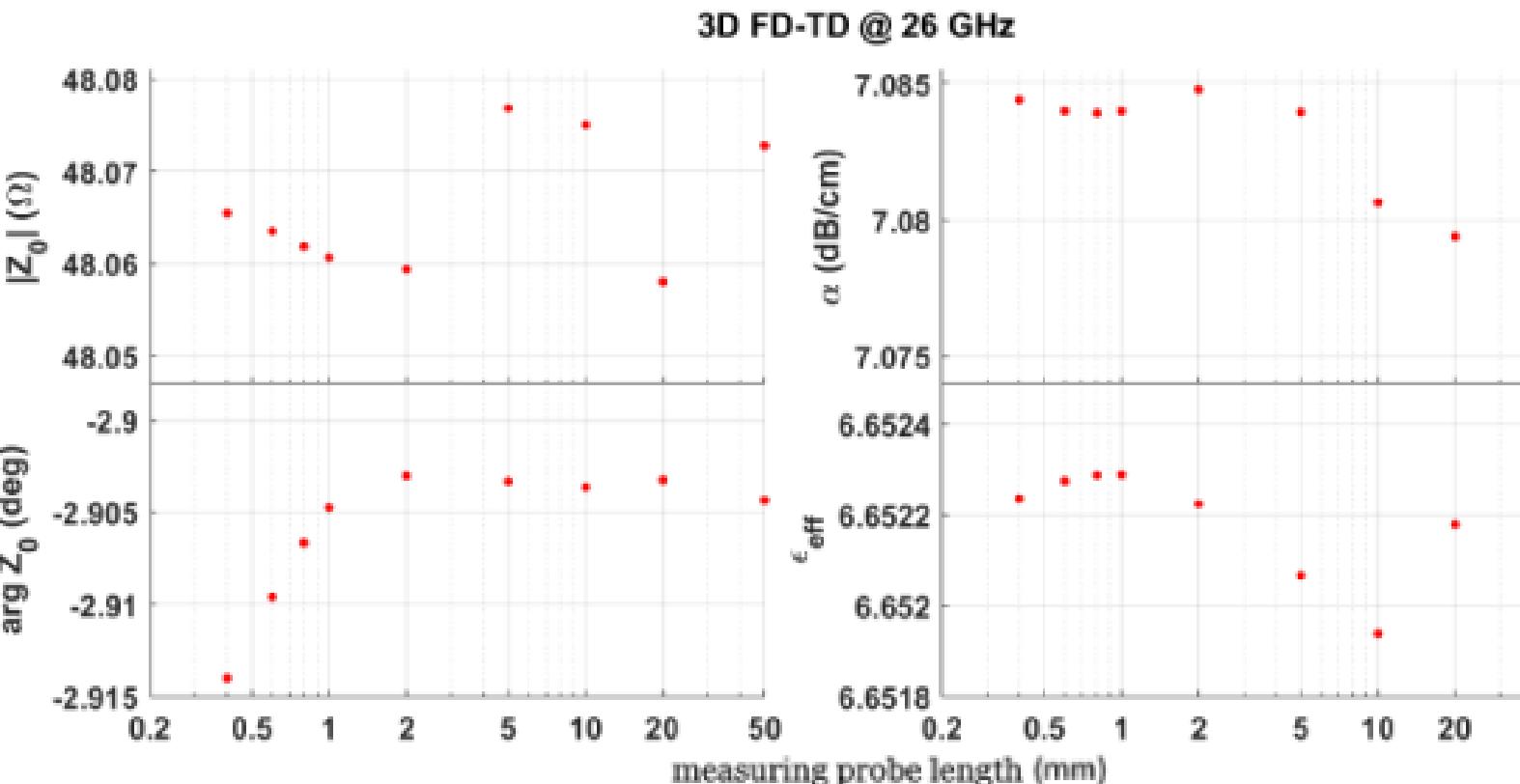
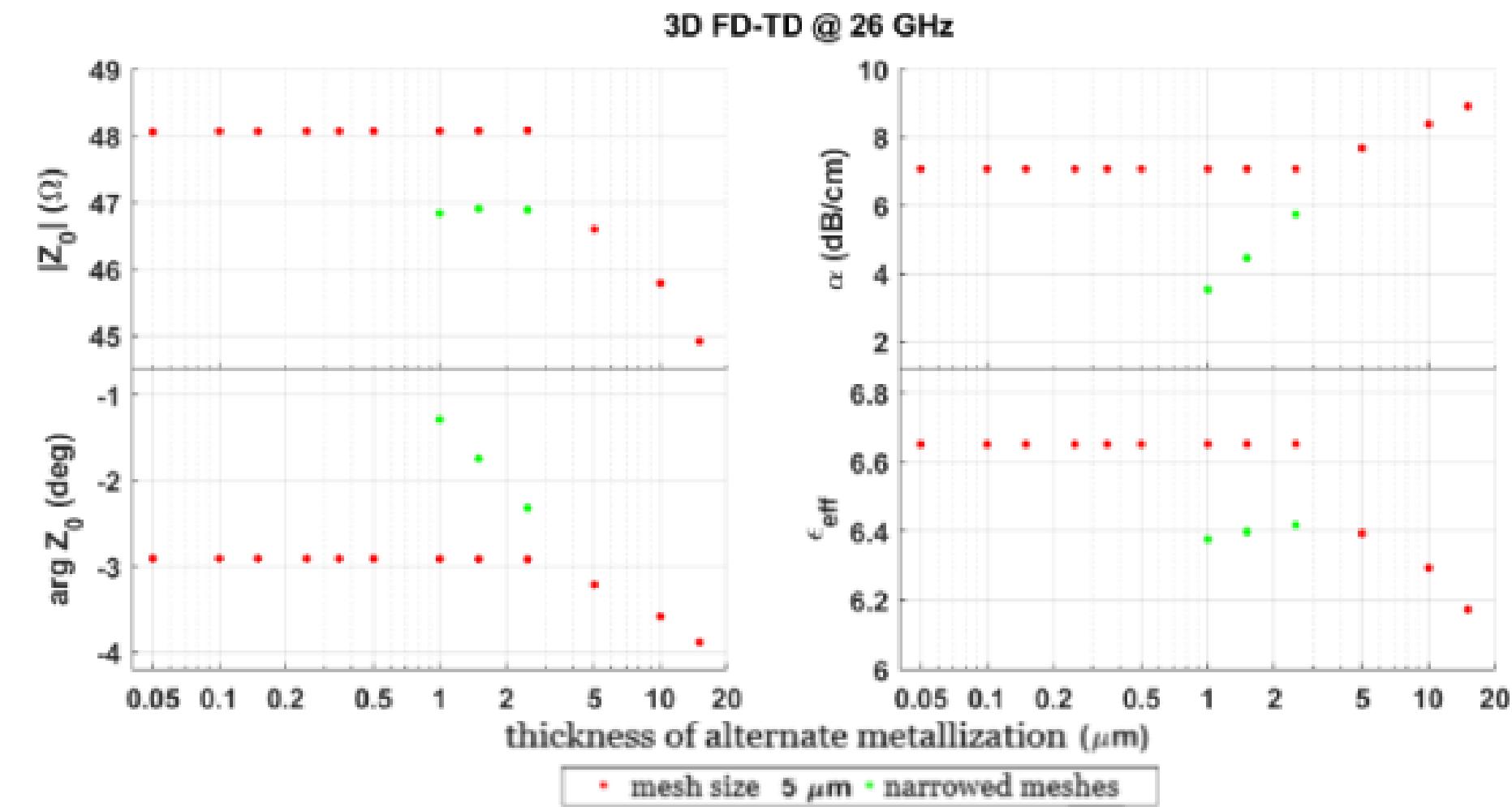
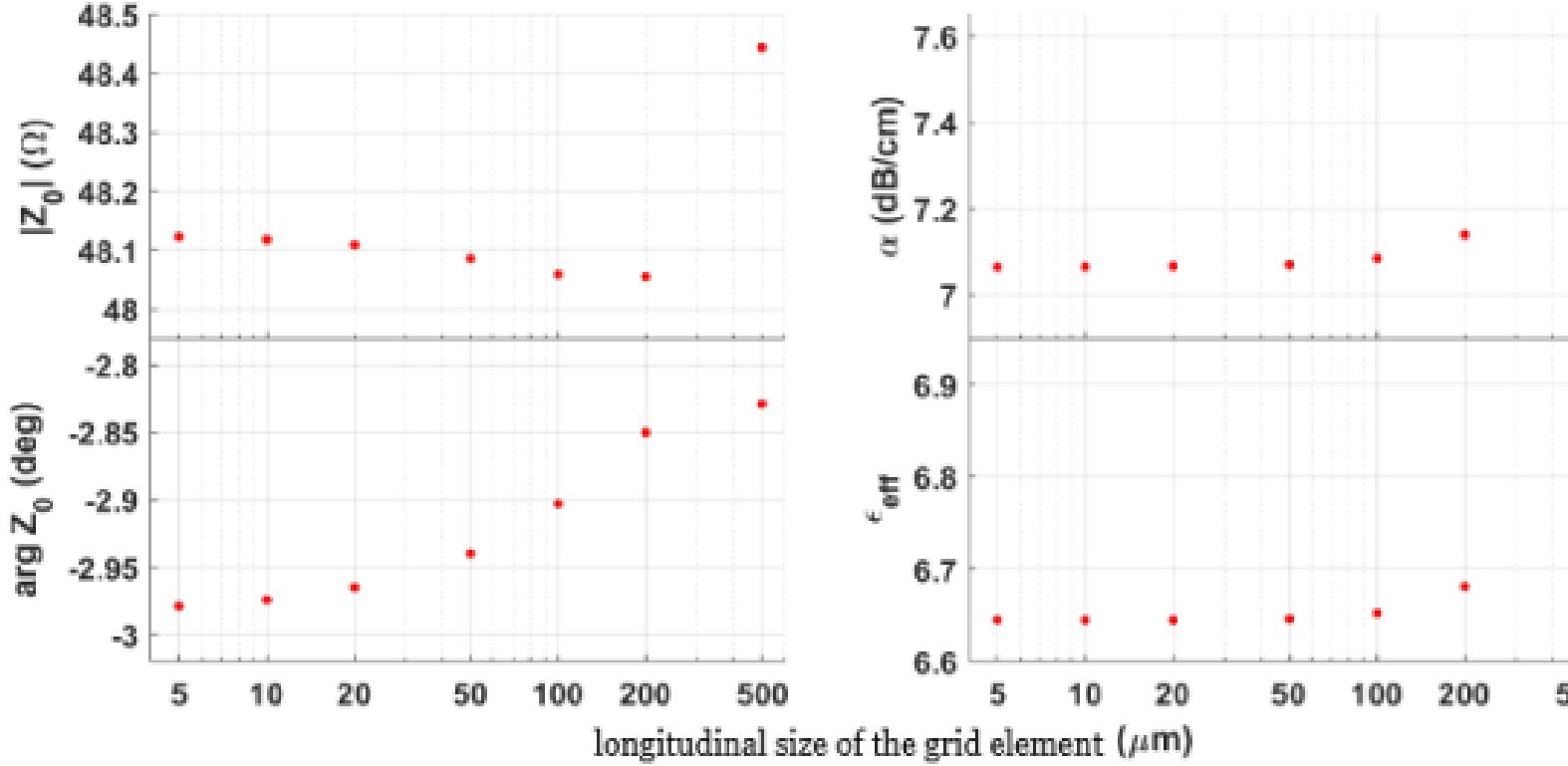


## Convergence tests

## Characteristic parameters

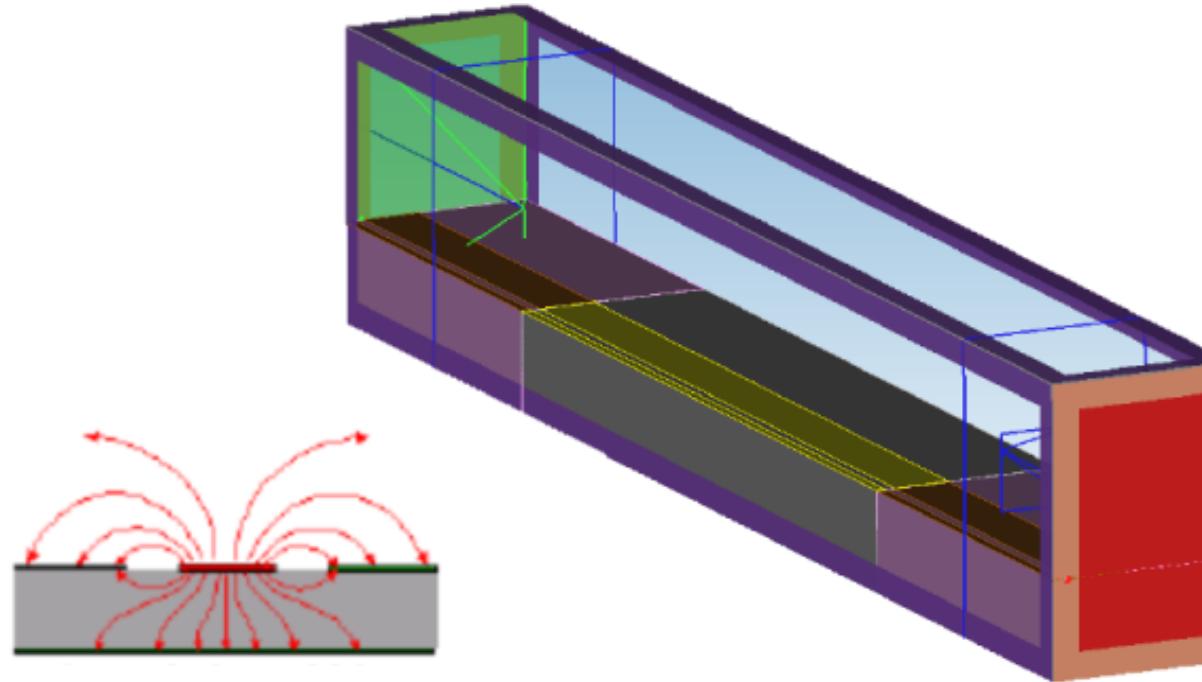


3D FD-TD @ 26 GHz



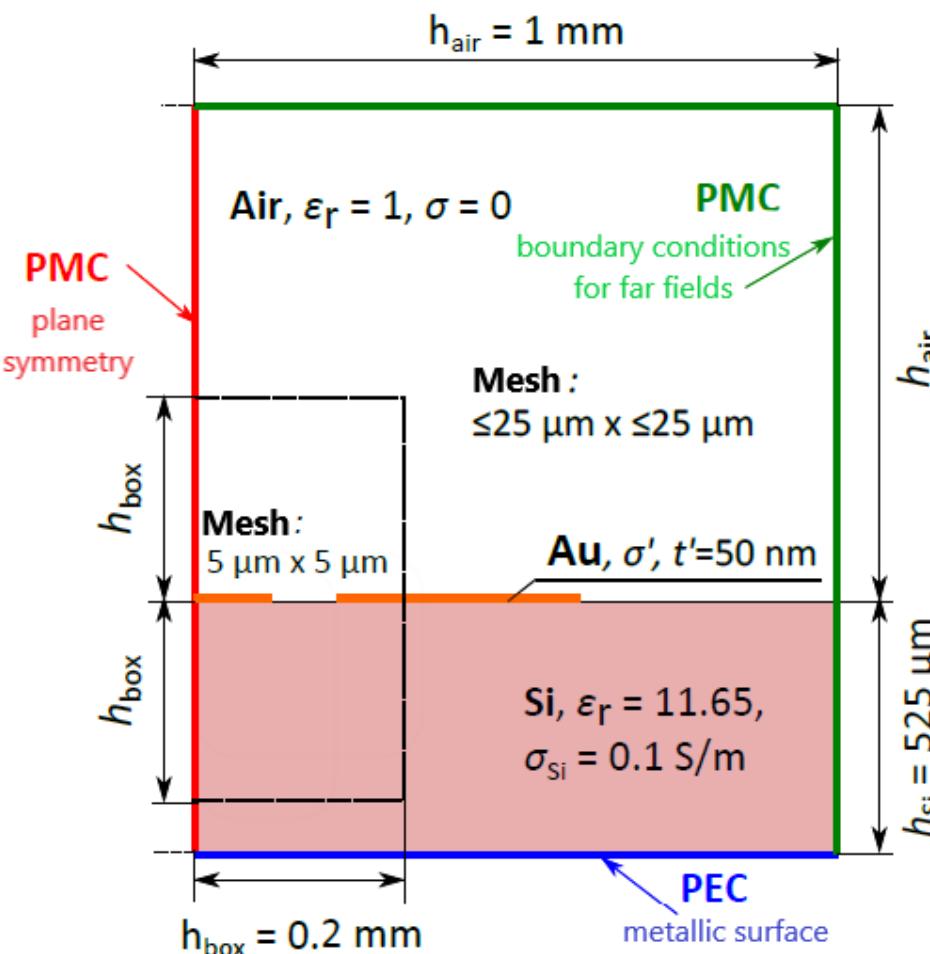
## Convergence tests

## Comparison

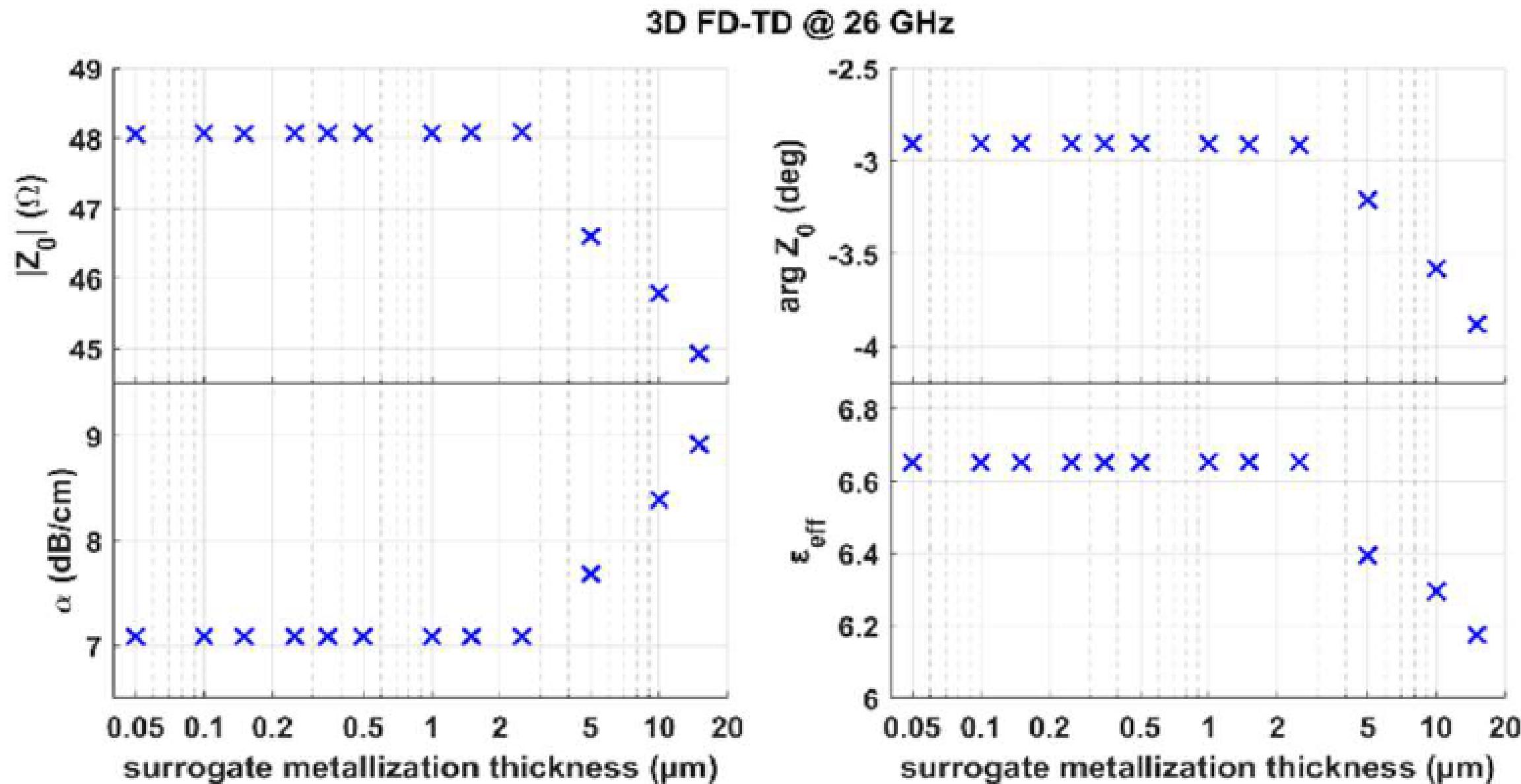


parameter	$\Delta S_{11} $	$\Delta \arg S_{11}$	$\Delta S_{21} $	$\Delta \arg S_{21}$
number of iterations	$\pm 0.000002$	$\pm 0.003^\circ$	$\pm 0.000003$	$\pm 0.0003^\circ$
space size $h_{\text{air}}$	-0.00002	-0.01°	-0.00005	-0.04°
transverse mesh size	—	10°	-0.04	—
size $h_{\text{box}}$	0.00003	$\pm 0.05^\circ$	$\pm 0.0002$	0.015°
longitudinal mesh size	-0.0002	-2°	-0.0007	-0.16°
measuring probe lenght	Fluctuations increase with the length of the probe			
thickness of alternate metallization	$\pm 0.0002$	$\pm 0.2^\circ$	$\pm 0.00003$	$\pm 0.0007^\circ$
frequency of the excitation pattern	-0.000005	-0.005°	-0.00005	-0.001°

parameter	$\Delta Z_0  (\Omega)$	$\Delta \arg Z_0 (\circ)$	$\Delta\alpha (\text{dB/cm})$	$\Delta\epsilon_{\text{eff}}$
number of iterations	$\pm 0.0002$	$\pm 0.0002$	$\pm 0.0001$	$\pm 0.00001$
space size $h_{\text{air}}$	0.02	$\pm 0.0005$	0.0015	0.0015
transverse mesh size	-1	-0.5	1	—
size $h_{\text{box}}$	-0.04	$\pm 0.0025$	$\pm 0.004$	-0.001
longitudinal mesh size	-0.07	0.08	0.02	0.007
measuring probe lenght	Fluctuations increase with the length of the probe			
thickness of alternate metallization	$\pm 0.012$	$\pm 0.0003$	$\pm 0.0005$	$\pm 0.00005$
frequency of the excitation pattern	-0.004	0.0007	0.0015	0.00005



# How does one efficiently model a layer of ultra-small thickness in general purpose EM simulators?



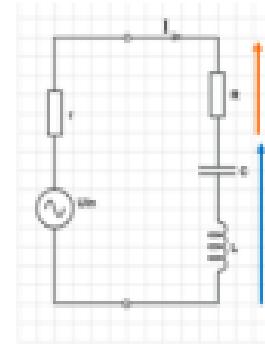
$$R_s = \frac{(d_0 \sigma_0)^{-1}}{(d \sigma)^{-1}}$$

parameters of the original metal

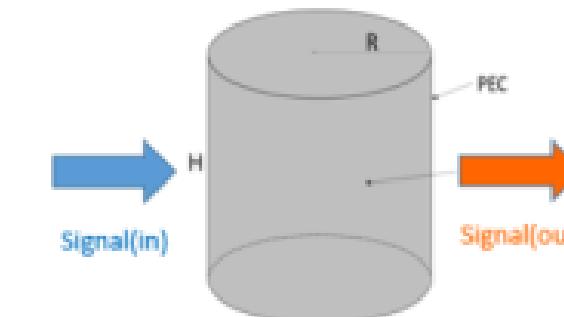
dielectric surrogate

Simulated parameters of CPW after [4] (case of  $W_2=300$  μm,  $R_s=0.326$  [Ω/□], thickness 100 nm Au + 5nm Cr) at 26 GHz, with uniform FDTD meshing of 5 μm, as a function of scaled surrogate thickness – in perfect agreement with the measurements of [4] for surrogate thicknesses below 2.5 μm and ca. 2% impedance error for thickness equal to the FDTD cell size.

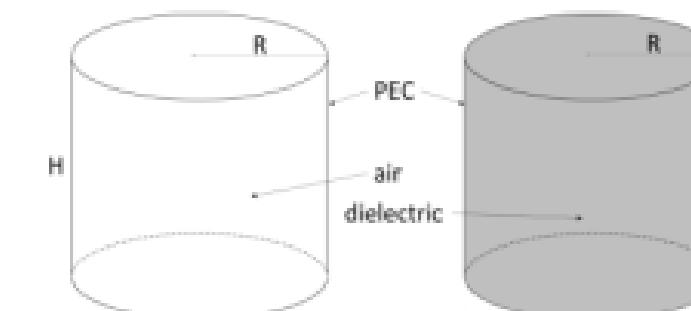
# Application of the model to development of a 10 GHz DR scanner for graphene anodes.



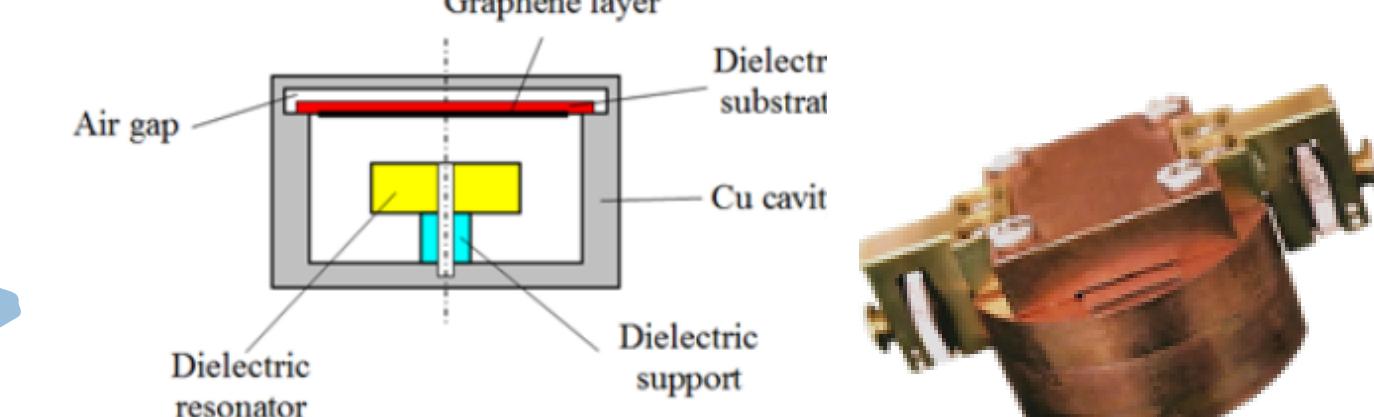
given fixed strength of  $U_{in}$ ,  
at resonance  $U_R$  is strongest ( $U_{LC}=0$ )



given fixed strength of **Signal(in)**,  
at resonance **Signal (out)** is strongest

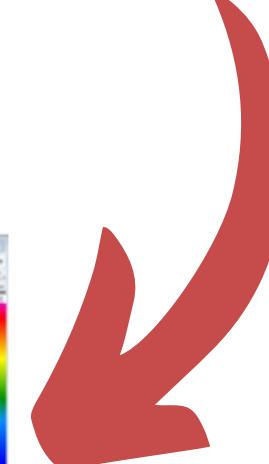
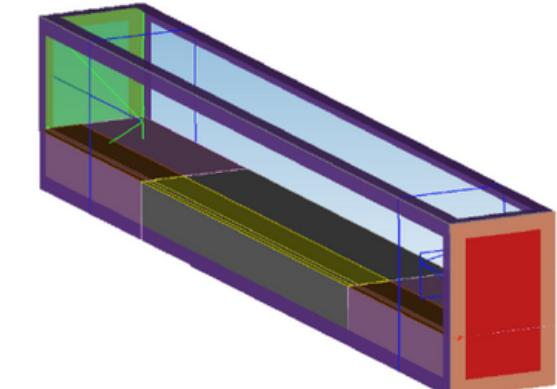
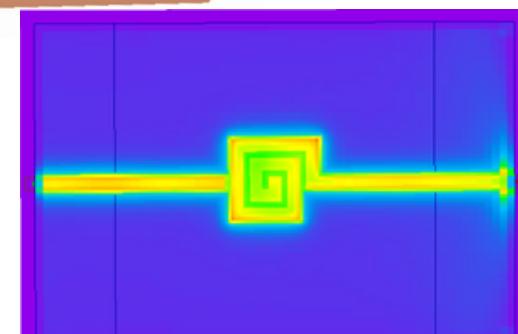
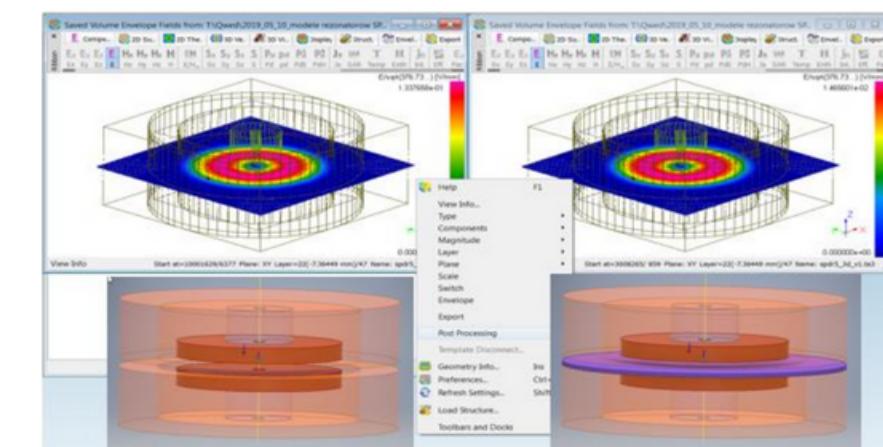
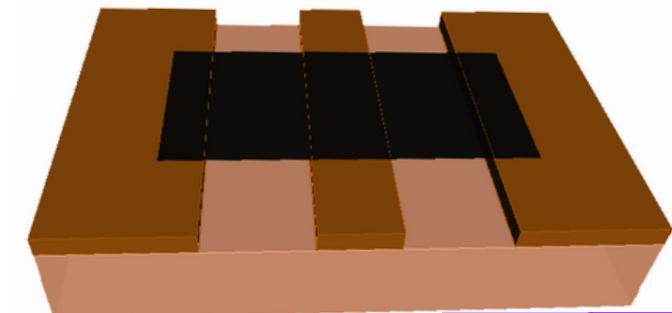
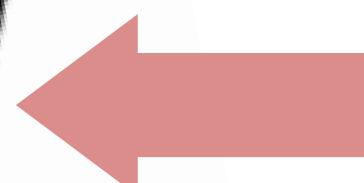
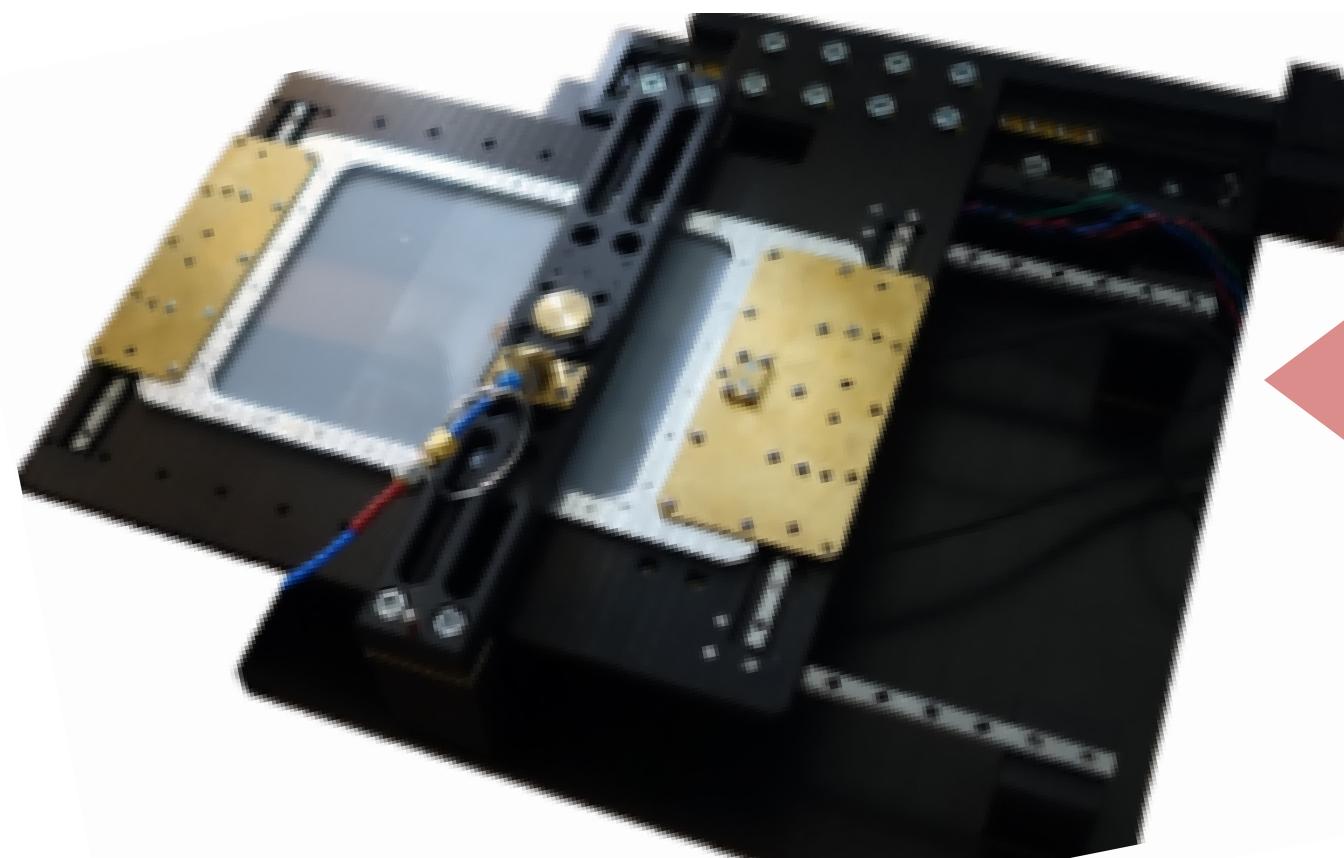
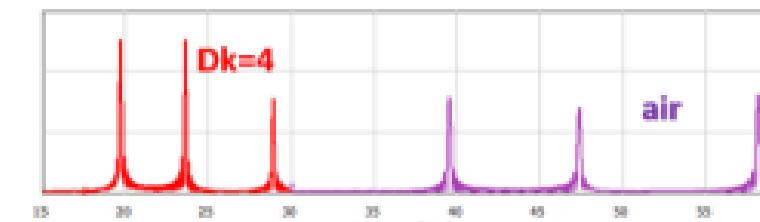
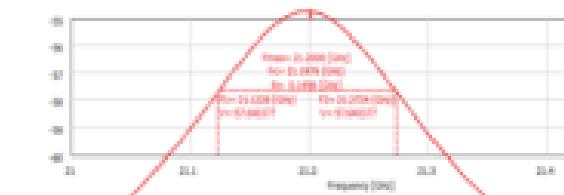


$$f_{r,amp} = \frac{c}{\sqrt{Dk}} \sqrt{\left(\frac{K_{mn}}{\pi R}\right)^2 + \left(\frac{p}{H}\right)^2}$$

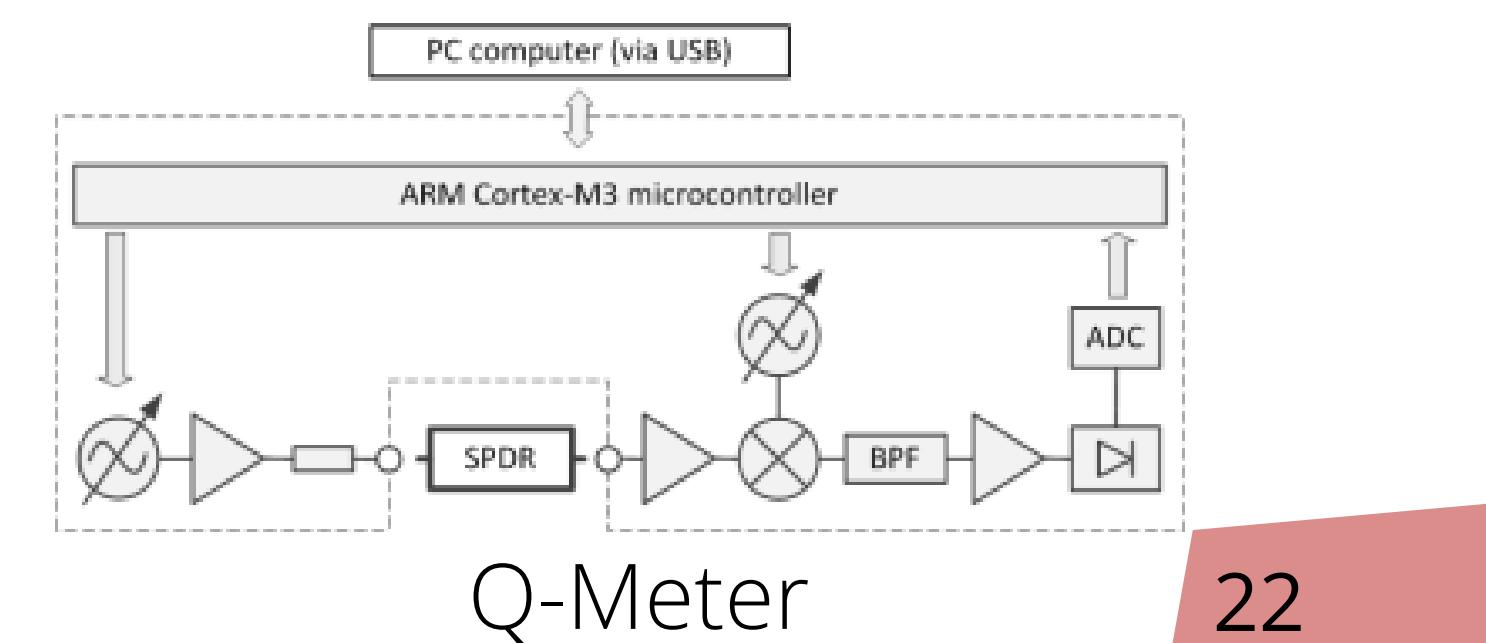
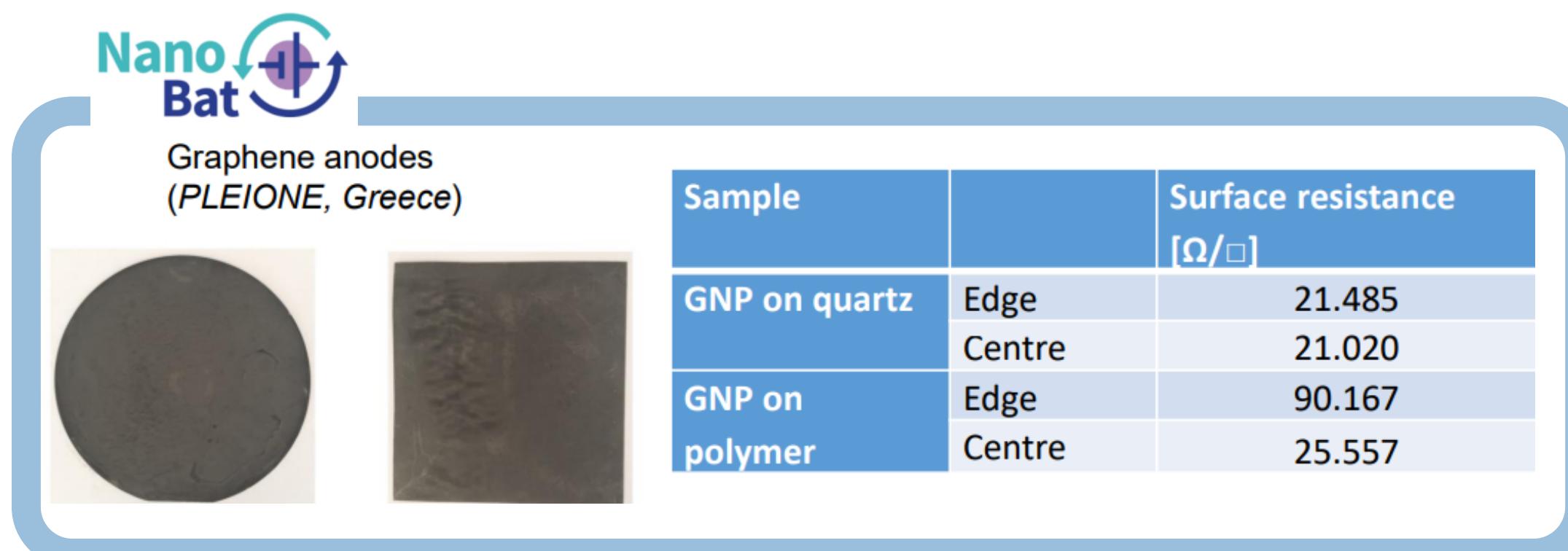
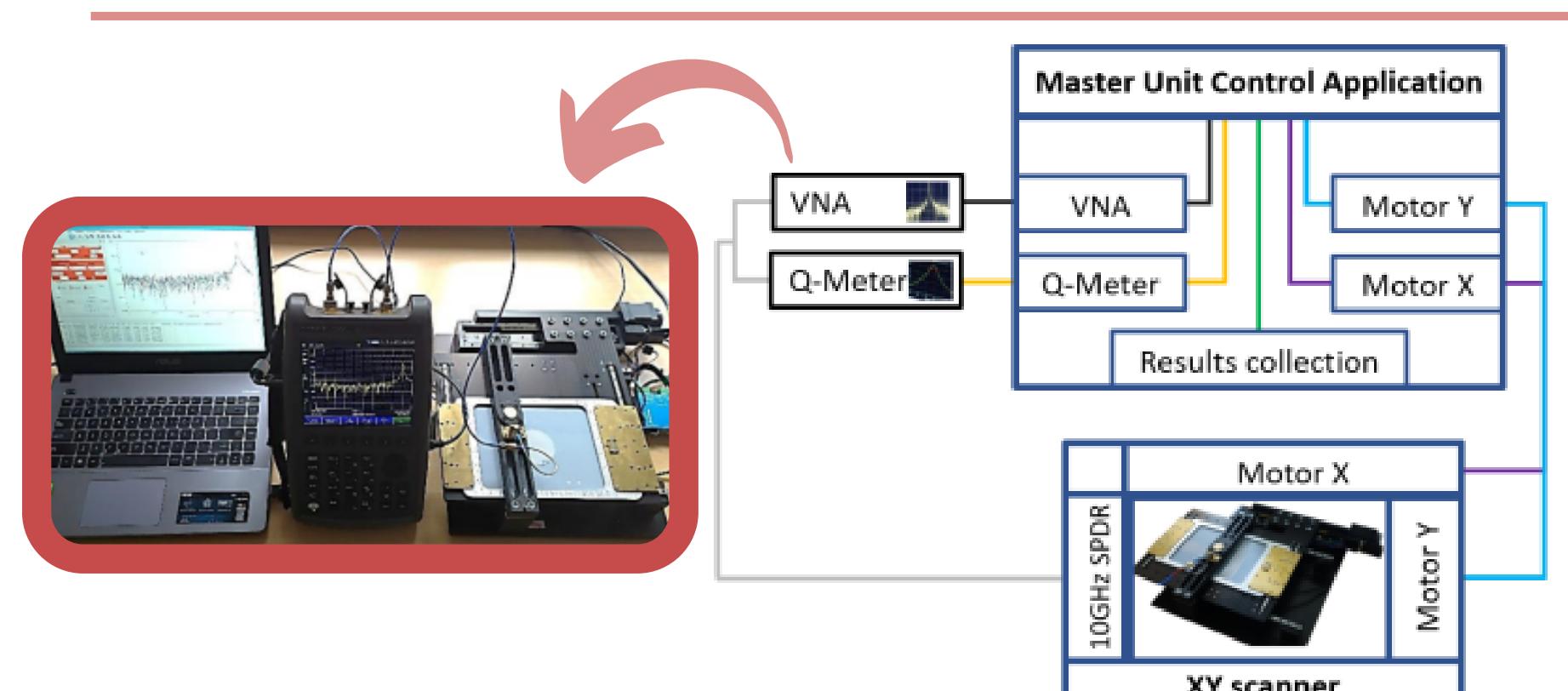
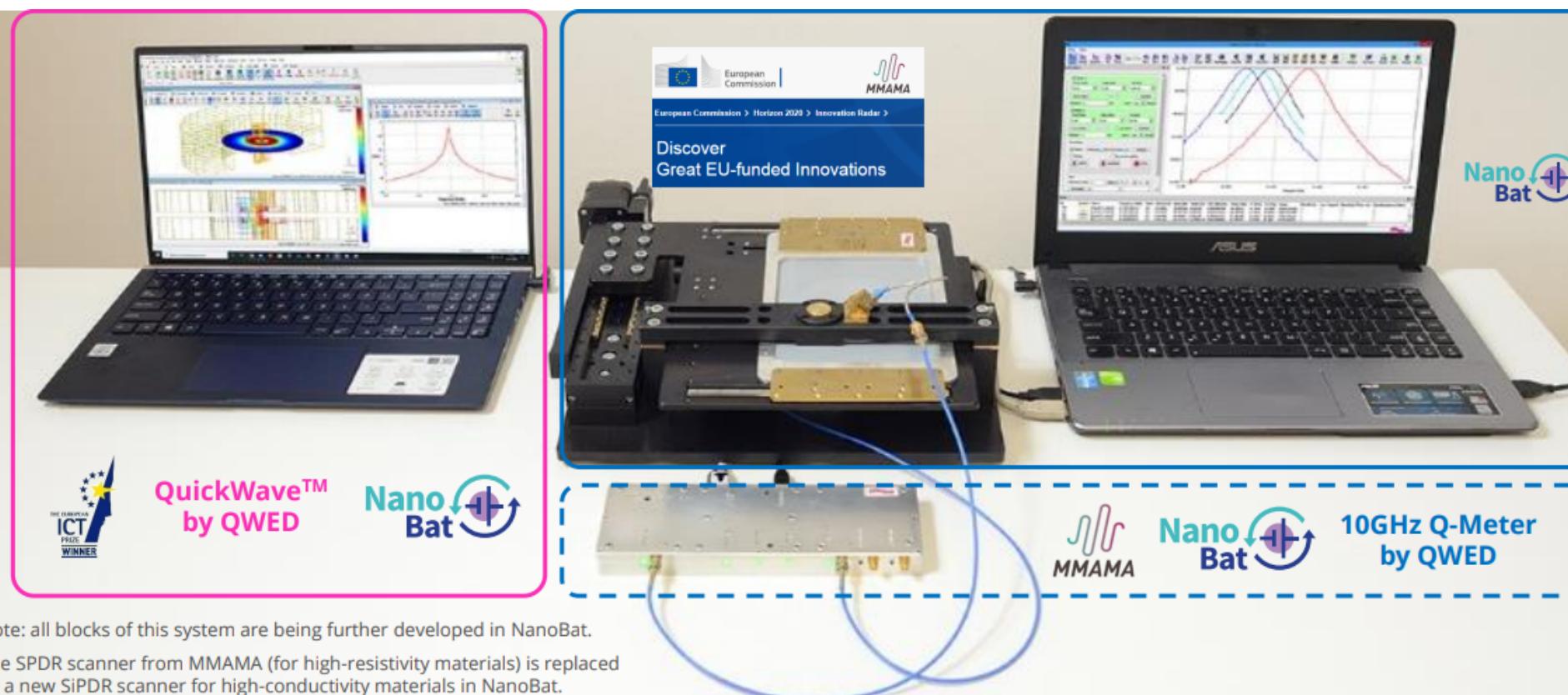


Cross section of the resonator for graphene measur

$$Q = 2\pi \frac{\iiint \epsilon \vec{E} \cdot \vec{E}^* dv}{T \iiint \sigma \vec{E} \cdot \vec{E}^* dv} = \frac{\omega \epsilon}{\sigma} = \frac{1}{Df} \approx \frac{f_{res}}{\Delta f}$$



# Application of the model to development of a 10 GHz DR scanner for graphene anodes.



# Conclusions

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In this work, simulations of a benchmark CPW structure with ultra-thin metalization made of gold thickness on a substrate of high-resistivity silicon for the frequency of 26 GHz

- Behind every **CHARACTERISATION** there is always a **MODEL**
- **Rs** parameter unambiguously determines the characteristic impedance, propagation constant, and effective permittivity of transmission lines with **ultra-thin** (<110 nm) signal and ground-plane layers.
- extensive study has been conducted for the convergence and accuracy of FDTD models of **CPW**-based material test-fixtures. For the thin films, a lossy dielectric **surrogate** preserving  $Rs = (d \sigma)^{-1}$  has been shown most efficient and valid up to the thickness of one **FDTD** cell (with cell size dictated by the **CPW** geometry and frequency range).
- Attenuation coefficient is directly and inversely dependent on the metallization thickness
- For more information about **DR** come today to Amphi 400C at **17:35**

# Acknowledgement

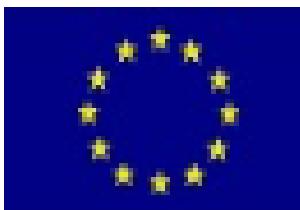
For authors:

Malgorzata Celuch & Marzena Olszewska – Placha & Konrad Wilczynski

The work presented has received funding from



NanoBat aims to develop a novel **nanotechnology** toolbox for quality testing of Li-ion and beyond **Lithium batteries**.



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# Thank You

