Accurate Analysis of Whispering Gallery Modes in Dielectric Resonators with BoR FDTD Method

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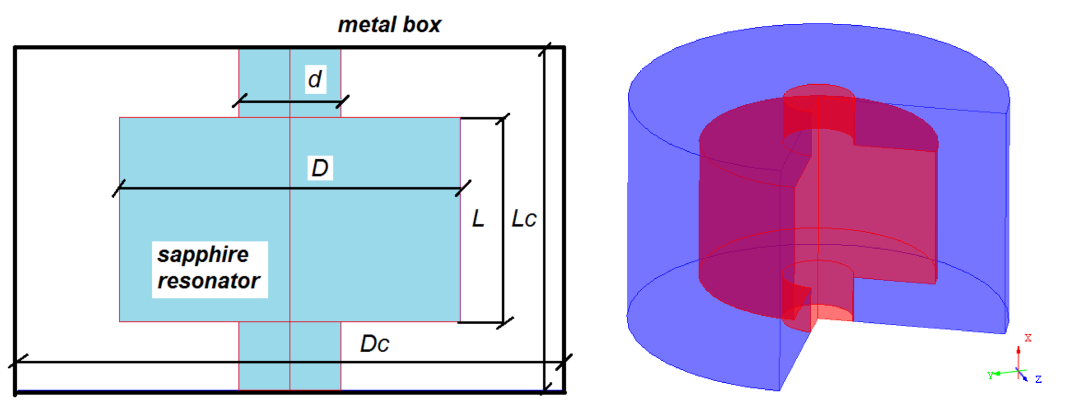
ABSTRACT This paper presents an accurate approach to FDTD analysis of whispering gallery modes in dielectric resonators. In those problems resonant frequencies are supposed to be extracted with relative errors below 10^{-4}. It is widely believed that only custom - made software codes, based on mode matching methods, can meet such stringent accuracy requirements. Herein, we demonstrate how the required accuracy can be obtained with a general - purpose FDTD code, run within a new three - step procedure. Advantages of the FDTD approach include more flexibility in modelling scenarios with unusual shapes or / and lossy materials.

The benchmark for this study

Sapphire disc resonator in a cylindrical metal enclosure



Resonator parameters after [1]:
sapphire relative permittivities are axial: 11.3532 transverse: 9.2747;
sapphire disc dimesions: D_c = 80 mm, L_c =50 mm;
cavity and support: D = 49.9894 mm, L = 30.008 mm, d = 15mm.

Accuracy requirement: better than 10^{-4}
for application in precise material measurements.

Previous numerical analysis approaches:
- custom-made codes
- in [1]: radial mode-matching with Rayleigh-Ritz technique
- accuracyprovided but limited flexibility in problem definition

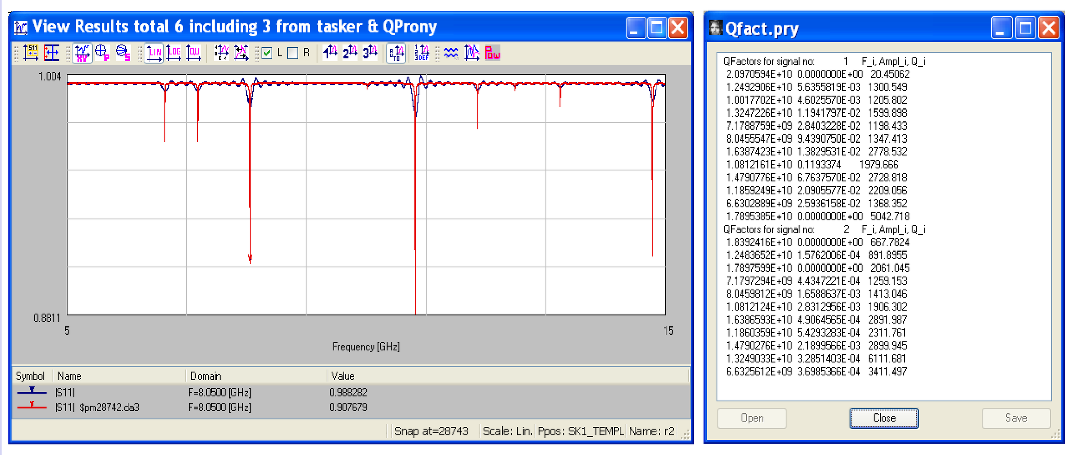
Why try to use FDTD?
+ general-purpose well-validated codes available
+ easy-to-use user interfaces
+ powerful co- and post-processing implemented
+ convergence properties generally understood
+ minor computer effort

Why the use FDTD has not been previously reported?
? alleged problems with meeting stringent accuracy requirements
? alleged problems with separating closely-spaced modes
? alleged problems with detecting "hybrid modes"

The aim of this study is to show that:
→ the previously "alleged" problems were due to inappropriate application of brute-force FDTD; not inherent in the method itself
→ BoR FDTD is an accurate approach to the analysis of whispering-gallery modes in dielectric resonators
→ our three-step procedure proves effective for detection and examination of modes

Applied three - step simulation procedure

Step1: mode detection
- launch Gaussian pulse 8..12 GHz,
- detect modes by GPOF method [5] in QProny [3],
- ultra-fast whole-band simulation <1min,
- convergence within $3*10^{-4}$ (oversampling > 256 ppp).



Step 2: accuracy enhancement
- launch narrow-band pulse around resonance,
- use DFT of [3],
- run-time 5-10 min per resonance,
- required accuracy reached:

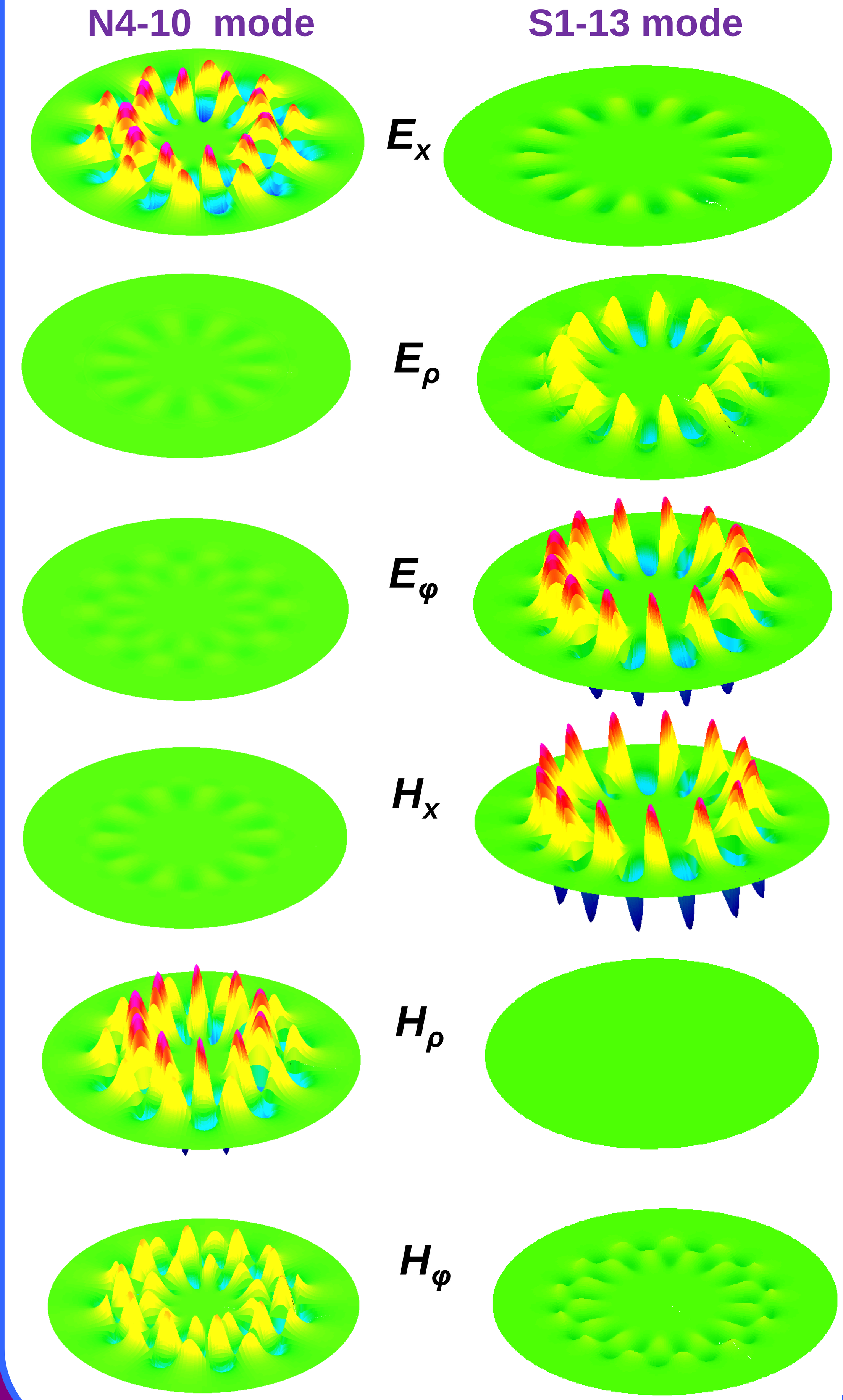
Mode - ang. var.	f_{E1} [MHz] exp. [1]	f [MHz] BoR FDTD	Δf [MHz] $f - f_{E1}$	relative error $10^4 \mid\Delta f\mid / f_{E1}$	f_{C1} [MHz] comp. [1]
N4 - 8	8512.81	8512.86	+ 0.05	0.06	8513.17
N4 - 9	9191.15	9191.22	+ 0.07	0.07	9191.34
N4 - 10	9864.02	9864.10	+ 0.08	0.08	9864.08
N4 - 11	10531.88	10531.90	+ 0.02	0.02	10529.60
N4 - 13	11855.00	11854.38	- 0.62	0.52	11852.06
S1 - 10	8217.60	8217.51	- 0.09	0.11	8218.69
S1 - 11	8805.50	8805.26	- 0.26	0.29	8806.33
S1 - 12	9395.60	9395.00	- 0.60	0.63	8396.13
S1 - 13	9987.20	9986.42	- 0.78	0.78	9987.64
S1 - 14	10580.00	10579.33	- 0.67	0.63	10580.31
S1 - 15	11173.80	11173.05	- 0.64	0.57	11173.89

→ relative errors <0.008% (absolute <0.8 MHz)
→ accuracy appropriate for the considered application
→ accuracy competitive to customised methods
→ errors less than expected from 3D FDTD error bounds

Step 3: investigating modal field patterns
- launch sine-wave at selected resonant frequency,
- use visualisation functionalities of QuickWave software (right).

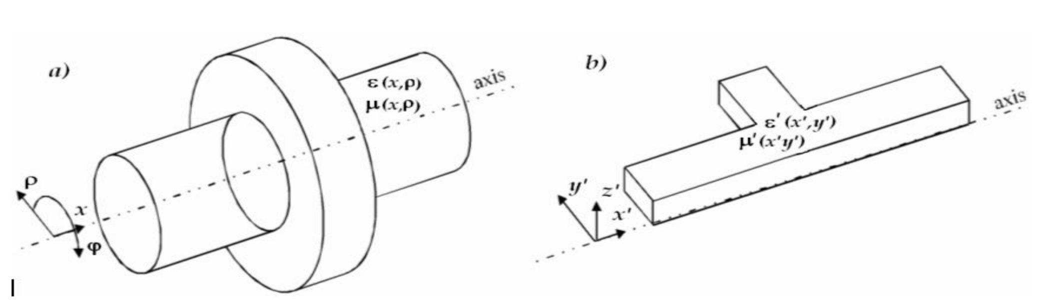
Field distributions of two example modes horizontal plane close to half-height of the resonator

N4-10 mode **S1-13 mode**



The concept of BoR FDTD

Reduce 3D axisymmetrical problem (a) to planar 2D (b):



Apply Maxwell equations in cylindrical coordinates:

- numerical FDTD discretisation in 2D plane ($x\rho \rightarrow x'y'$)
  → economies in computer effort by 2-3 orders in magnitude

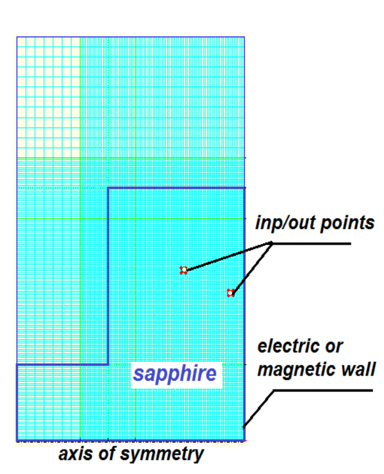
- angular cos($m\varphi$) / sin($m\varphi$) field dependence enforced analytically
  → expected higher accuracy for high-m modes

Discretisation and computer requirements:

- from general FDTD error bounds: 50 cells per wavelength needed
- we use 0.2 mm in sapphire, relaxed to 0.5 mm in air
- set symmetry plane at half-height of the resonator
- total of ca. 16 000 FDTD cells
- ca. 2 000 FDTD iterations per sec on a standard laptop

Excitation and parameter extraction:

- virtual source (input) needed to launch FDTD simulation
- probe (output) added for mode detection
- input & output position & internal impedance help distinguishing modes
- DFT or GPOF co-processing used for detecting resonances



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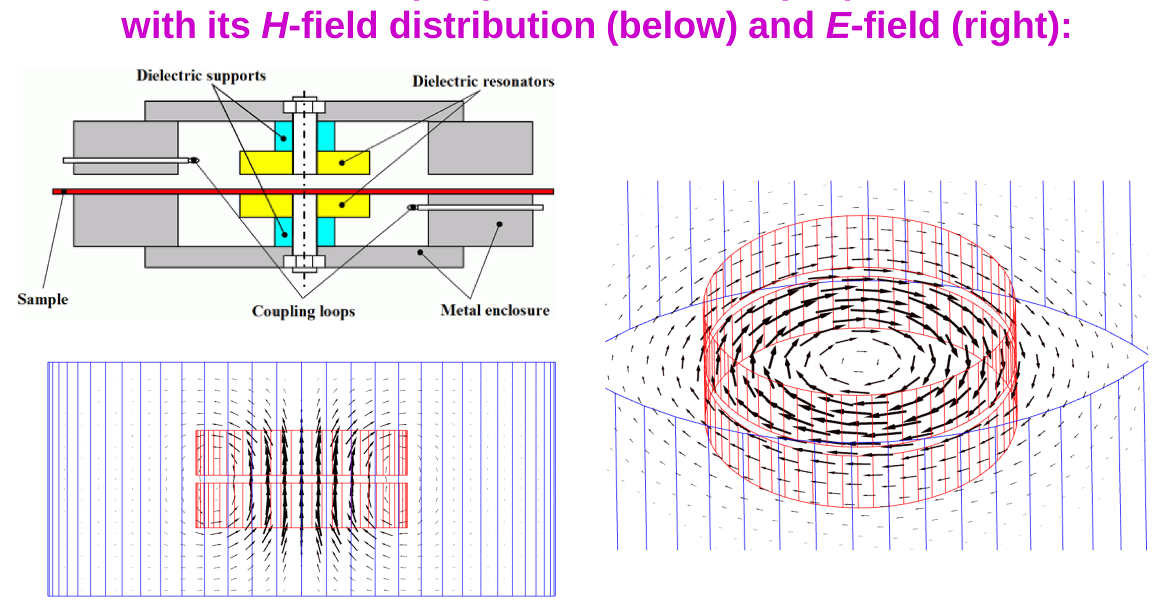
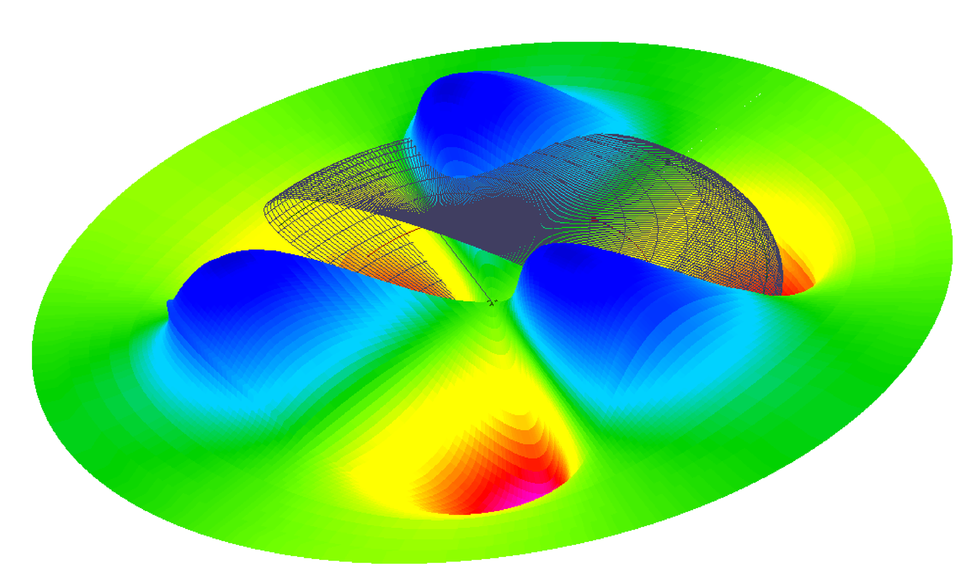
Related applications of the proposed BoR FDTD procedure

Look-up tables for material measurements in split-post resonators (left) with its *H*-field distribution (below) and *E*-field (right):



Analysis of modes in ellipsoidal dielectric resonators example E_φ field across the resonator for the lowest mode with angular variation 3:



Prospects for further developments

1. Development of GPOF method after [5] for accurately postprocessing signals oversampled at more than 256 points per period. This will accelerate the extraction of resonant frequencies by curtailing step B of the procedure.

2. Application of BoR FDTD to evalute the effects of imperfect cylindricity of practically manufactured samples (non - flat top and bottom surfaces, conical side walls or chamfered edges) and metal losses on the results of complex permittivity measurements.

3. Rigorous derivation of error bounds for BoR FDTD, following the characteristic equation approach previously developed in Cartesian coordinates [4] and taking into account dielectrics [6]. This will facilitate mesh optimisation for desired accuracy.

References:

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