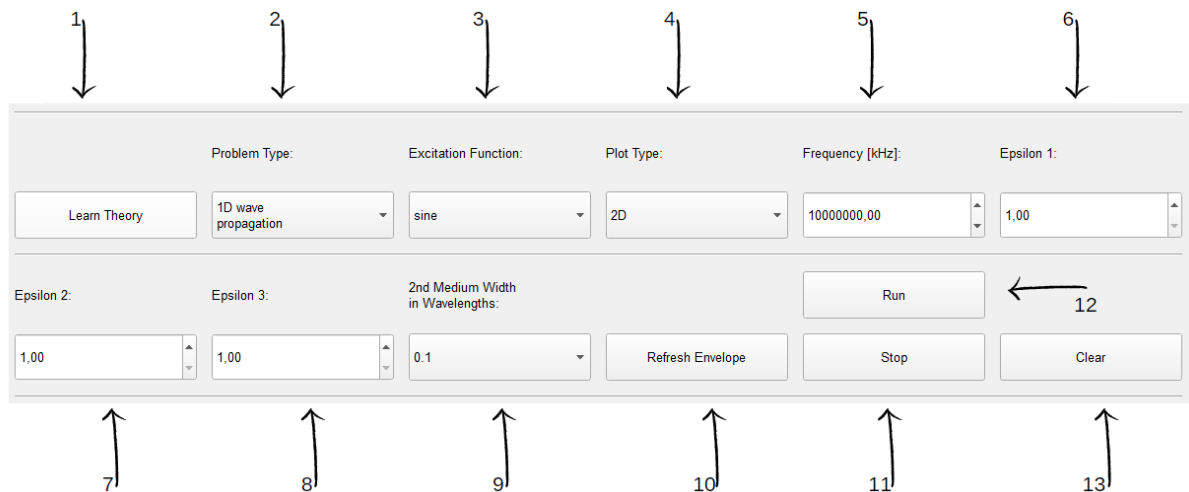# Manual and Theory

1. Application layout

Figure 1.1 Application layout

1. *Learn theory* – this button opens a new window with a present "Manual and Theory" file

2. *Problem type* – allows for a definition of a problem type. There are two options to choose from:

- 1D electromagnetic wave propagation
- 2D electromagnetic wave propagation

3. *Excitation function* – describes a signal function which will be generated by the signal source. There are two options to choose from:

- sine
- gauss (Gaussian pulse)

4. *Plot type* – allows for choosing plot/display type, which will be used to display simulation results. There are two options to choose from:

- 2D
- 3D

5. *Frequency* – wave frequency, which can be changed by keyboard input or using spin box

6., 7., 8. *Epsilon* – value of a relative permittivity of medium 1, 2, and 3 respectively:
- options 7 and 8 are available only for 1D EM wave propagation problems
- in the case of 1D EM wave propagation problems, *Epsilon 1* parameter should be set to 1

9. *2nd Medium Width in Wavelengths* – width of area bounded by two blue lines indicated in Figure 2 (available only for 1D EM wave propagation problems), filled with medium of *Epsilon 2*, expressed in wavelengths of *Frequency*.

10. *Refresh Envelope* – pressing this button starts drawing a new envelope

11. *Stop* – this button stops the visualisation of simulation results

12. *Run* – this button starts the visualisation of simulation results

13. *Clear* – this button clears the window with visualisation of simulation results
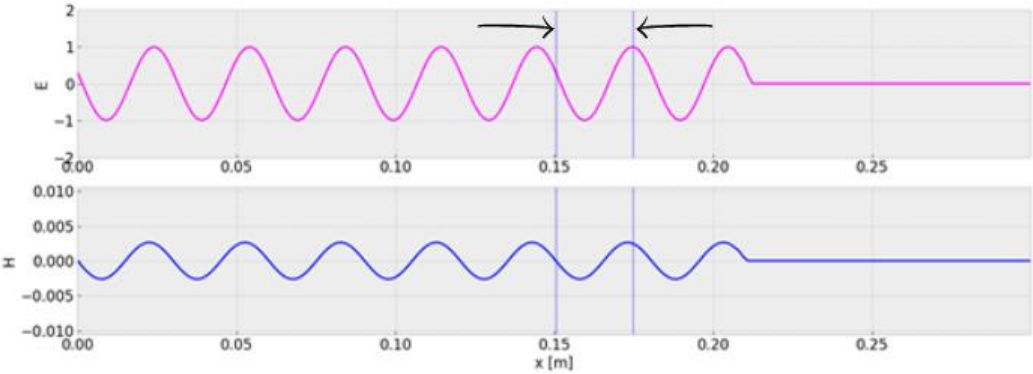

Figure 1.2 1D EM wave propagation – half-space filled with medium of Epsilon 2 marked with blue lines (width of that area is expressed in wavelengths)

2. Theory and Examples

The electromagnetic field is considered one of the fundamental phenomena in nature, as it plays a major role in determining the internal properties of most objects encountered in daily life. Many different physical forces are derived based on electromagnetic field and a significant part of new technologies is developed owing to extensive research on electromagnetic interactions. Having a general knowledge of the electromagnetic field allows us to better understand the reality around us. Several important aspects of electromagnetism, more precisely the propagation of electromagnetic waves, will be discussed in the following sections.

2.1 Electric and magnetic fields around conductor

We know that current flow through an infinite wire produces a magnetic field, which direction can be determined by using the "right-hand rule". We can illustrate the magnetic induction (B) changes on a plot as presented in Figure 1 - sinusoidal current flow produces sinusoidal changes of B.
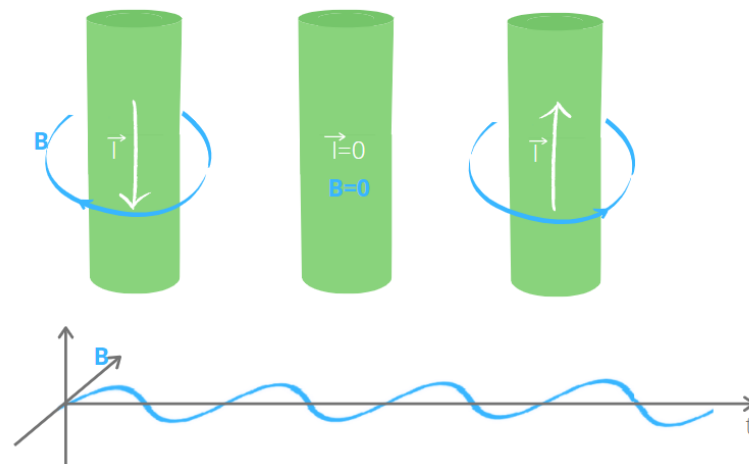

Figure 2.1 Magnetic field (B) changes in time

The changing B produces an electric field (E). The vectors of B and E are perpendicular to each other.

Figure 2.2 Magnetic and electric fields changes in time

On the plots presented above we can see E and B values for certain location near the wire, which are changing in time. What happens if we move away from our conducting wire or if we switch the electric current source off?

The combination of disturbances of the electric and magnetic fields will propagate in the direction perpendicular to the fields oscillations. In that way an electromagnetic wave is created and its propagation is highly dependent on the medium in which it propagates. There are many different

applications of electromagnetic waves, without it we would not be able to develop advanced medical diagnostics, heat up food in the microwave or use Wi-Fi in our smartphones. Some characteristic parameters and equations describing the electromagnetic waves will be presented in the following sections to deliver a background insight into a phenomena which surrounds us.

1.2. Characteristic parameters and equations

The characteristic parameters of an electromagnetic wave are: wavelength, frequency, magnitude, propagation speed.

These parameters are related to each other by the following formula:

$$f = \frac{v}{\lambda}$$

where:

- λ - wavelength,
- f - frequency
- v - propagation speed.

Propagation speed is highly dependent on medium parameters, what is indicated by the following formula:

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

where:

- μ - medium permeability, which gives us information about the resistance of a medium against the formation of a magnetic field, it is measured in henries per meter (H/m), μ_0 - permeability of vacuum, μ_r – relative permeability (material property)),
- ε - medium permittivity, which tells us how easily can we electrically polarize a dielectric, materials with higher permittivity polarizes more in response to an applied electric field than materials with lower permittivity, ε_0 – permittivity of vacuum, ε_r – relative permittivity (material property),
- c – speed of light in vacuum.

The properties of the electric and magnetic fields and the relationship between these fields are mathematically described by Maxwell's equations:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

(Faraday's law - A time-varying magnetic field creates an electric field.)

$$\nabla \times \boldsymbol{B} = \mu\boldsymbol{j} + \mu\varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

(Ampere's law - The flowing current and the alternating electric field create a magnetic field.)

$$\varepsilon\nabla \cdot \boldsymbol{E} = \rho$$

(Gauss's law for electric field - Charges are a source of an electric field.)

$$\nabla \cdot \boldsymbol{B} = 0$$

(Gauss's law for magnetism - The magnetic field is source less.)

where

- j is electric current density,
- ρ is electric charge density.

The propagation of the electromagnetic wave, which is presented in this instruction is mathematically described by Faraday's law and Ampere's law. In the equations presented above, we can see **H** (A/m) vector instead of **B** (T). In our case, these vectors are related to each other as follows:

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

Permittivity and permeability can take different forms, e.g. they can depend on frequency (dispersive) or the value of the applied field (nonlinear). In our case we are considering a simplified model of nondispersive, linear and isotropic material, so µ and ε are scalars.

1.3. 1D problems

The application can be used to visualise simplified 1D and 2D electromagnetic wave propagation. In 1D mode there is a possibility to visualise wave propagation through 3 different media. This corresponds to everyday life examples, such as radio/mobile phone/WiFi signal crossing house wall or window (Figure 3). We assume a simplified model in which our signal is propagating in an isotropic medium without losses and incident normally at the media boundaries. In our 1D space there are variations only in one direction, let us assume it to be X. Let's also assume that our **E** vector has only one nonzero component – Ez.

$$\nabla \times \boldsymbol{E} = a_x \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + a_y \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + a_z \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)$$

$$\nabla \times \boldsymbol{E} = a_x (0 - 0) + a_y \left(0 - \frac{\partial E_z}{\partial x} \right) + a_z (0 - 0)$$

In this case, the Faraday's law equation simplifies to:

$$\frac{\partial Ez}{\partial x} = \mu \frac{\partial Hy}{\partial t}$$

and accordingly, the Ampere's law equation takes the following form:

$$\frac{\partial Hy}{\partial x} = j + \varepsilon \frac{\partial Ez}{\partial t}$$

This two coupled equations where used in order to prepare a visual simulation of wave propagation in space and time.
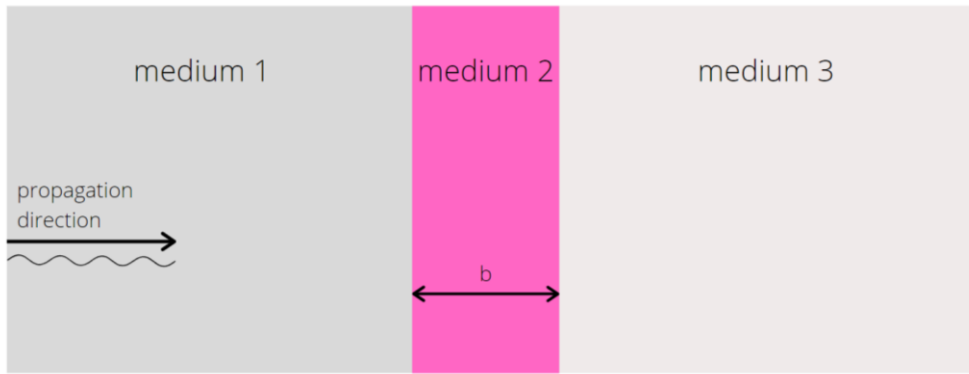
Figure 2.3 Wave propagation through different media

In order to visualize the situation described above (Figure 3) we have to set proper *Problem Type* (1D wave propagation) and change parameters (e.g. *Epsilon 1* and *Epsilon 3* can be set to 1, which is equal to air permittivity, *Epsilon 2* can be set to 3.7, which is equal to brick permittivity and *Frequency* can be set to 10000000kHz (10GHz).
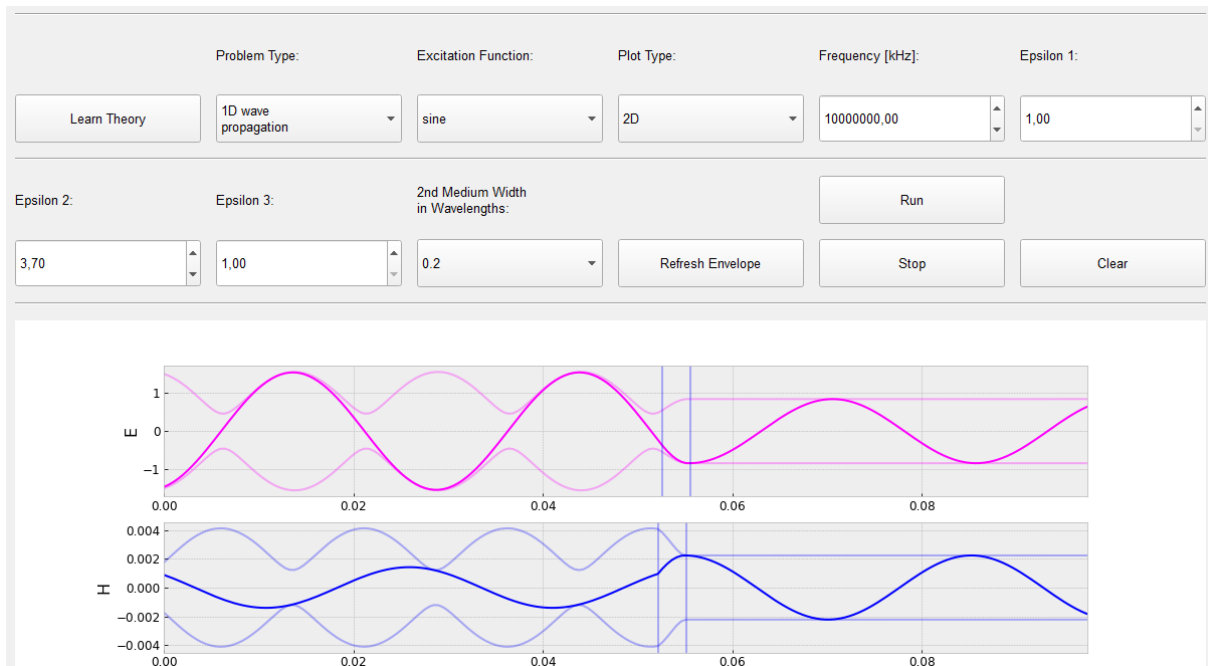

Figure 2.4 1D simulation for ε_2=3.7 and 2nd medium width=0.2wavelength

It is also possible to plot E and H values on 3D plot. This option is available after changing *Plot Type* to "3D" (Figure 5).
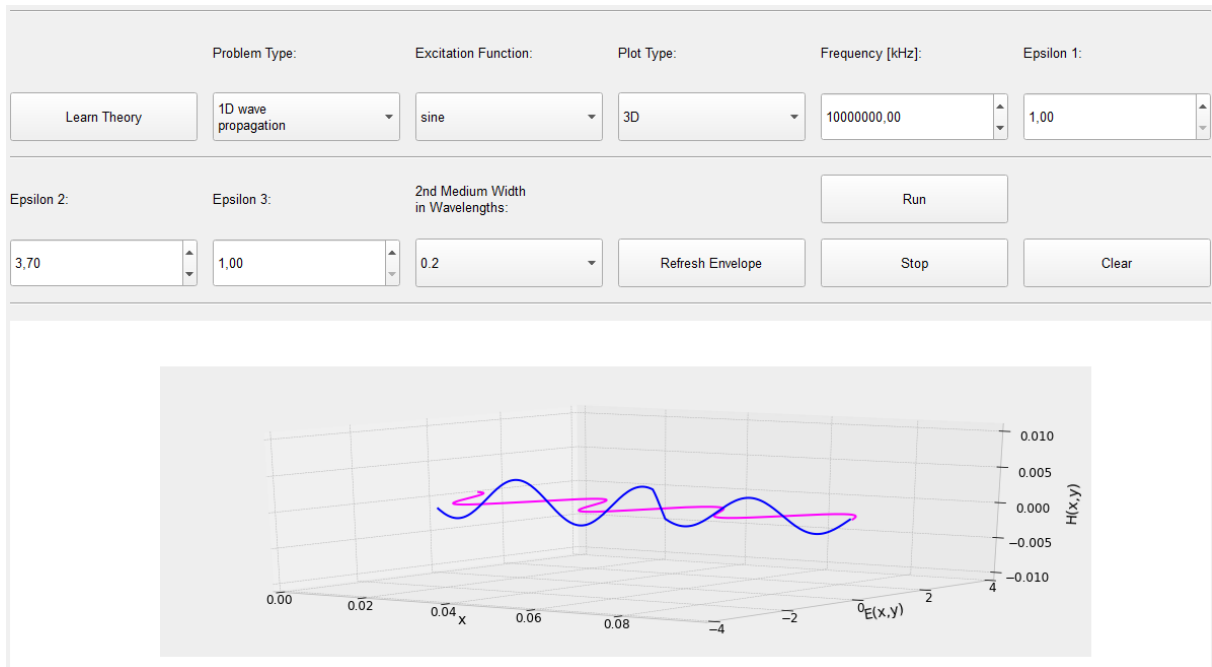
Figure 2.5 1D simulation for ε_2=3.7 and default 2nd medium width, 3D plot

During simulation we can observe that on the media boundary some part of incident wave is transmitted to the second medium and some part is reflected back towards the signal source. It can also be seen that incident and reflected waves merge, giving partially standing wave. The reflection coefficient can be calculated from the formula presented below:

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where Z_2 and Z_1 are characteristic impedances of medium 2 and 1 respectively and:

$$Z_n = \sqrt{\frac{\mu_n}{\varepsilon_n}}$$

Negative value of the reflection coefficient means that the wave propagates from a medium of higher impedance.

Another useful coefficient, which describes a wave behaviour at the media boundary is a Standing Wave Ratio (SWR):

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

The SWR takes values from the interval $(1; \infty)$. For small values of SWR (tending to 1), we deal with a more traveling-like wave.

In the present application, we can manipulate the value of reflection coefficient/SWR by changing *2nd Medium Width in Wavelengths* (our wall).

In the case of plane wave incident on a media stack, we should consider a more complex formula for an impedance "seen" by a wave incident at media 1 and 2 boundary. The plane wave that incidents at the boundary of medium 1 and medium 2 is partially transmitted to medium 2. Then this partially transmitted wave incidents at the boundary of medium 2 and medium 3. Again, a part of the wave is

transmitted to medium 3 and another part is reflected and propagates in medium 2. Thus, the wave that propagates in the second medium is influenced both by the wave reflected from the boundary of medium 2 and medium 3 and wave transmitted from the boundary of medium 1 and medium 2. In order to simplify the calculation of reflection coefficient or SWR at the boundary of medium 1 and medium 2 we can define an adjusted impedance, which takes into account the wave reflected from the boundary of medium 2 and medium 3:

$$Z_{2a}(b) = Z_2 \frac{Z_3 - jZ_2 tg(\beta_2 b)}{Z_2 - jZ_3 tg(\beta_2 b)}$$

where b is the second medium width (Figure 3).

For specific widths of our wall we can obtain impedance matching of different media at the boundary of medium 1 and 2, which means that reflection is minimized and no/almost no standing wave is created. The impedance matching is desirable e.g. in systems containing transmission lines. If the impedances at each end of the line are not matched to the transmission line's characteristic impedance the reflections of the signal at the ends of the line are generated. In general, impedance matching is desirable to ensure the best possible signal transmission efficiency.

There are two special cases of impedance matching:

- using half wavelength transformer
- using quarter wavelength transformer

In both cases, our goal is to obtain $\Gamma = 0$ that results in $SWR = 1$.

1.2.1. Half wavelength transformer

In the case of half wavelength transformer, the thickness of medium 2 needs to be equal to half of wavelength propagating in this medium (b=$\frac{\lambda_2}{2}$) and therefore, we obtain:

$$tg(\beta_2 b) = tg\left(\frac{2\pi}{\lambda_2}\frac{\lambda_2}{2}\right) = tg(\pi) = 0$$

and

$$Z_{2a}(b) = Z_3$$

If

$$Z_3 = Z_1$$

then the reflection coefficient at the boundary of medium 1 and medium 2 equals:

$$\Gamma_{1,2} = \frac{Z_3 - Z_1}{Z_3 + Z_1} = 0$$

We can check the operation of half wavelength transformer in the application, using our previously considered example with brick wall. Comparing a scenario with default value of *2nd Medium Width in Wavelengths* (0.2) (Figure 4) with a half wavelength width (0.5) (Figure 6), we can see a significant difference in the wave shape (remember to refresh the envelope, when the system is stabilised).
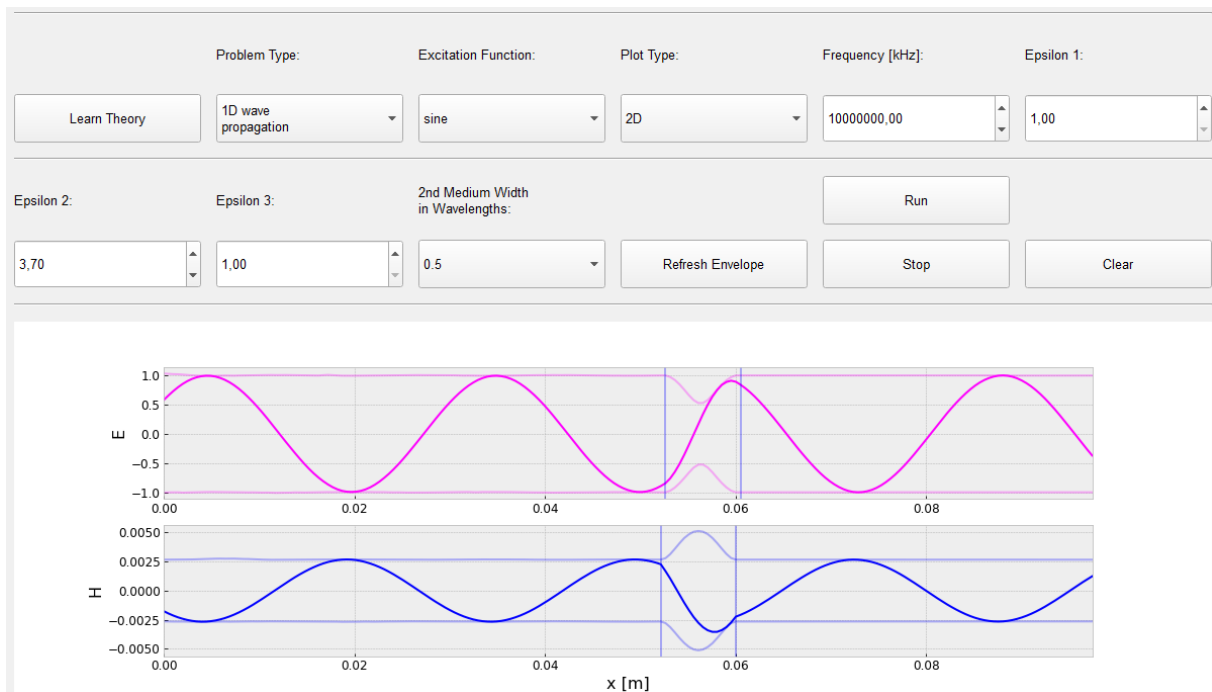
Figure 2.6 Wave propagation - half wavelength transformer

Due to the fact that tangent function is periodic with period equal to π, the thickness of medium 2 can be equal to $n\frac{\lambda_2}{2}$, where n is an absolute positive number, and the same effect will be obtained (only the transient state will last longer for higher n).

1.2.2. Quarter wavelength transformer

We can obtain similar results using quarter wavelength transformer. In this case b=$\frac{\lambda_2}{4}$ and therefore we obtain:

$$tg(\beta_2 b) = tg\left(\frac{2\pi}{\lambda_2}\frac{\lambda_2}{4}\right) = tg\left(\frac{\pi}{2}\right) = \infty$$

and

$$Z_{2a}(b) = \frac{Z_2^{\,2}}{Z_3}$$

If

$$Z_2 = \sqrt{Z_1 Z_3}$$

then the reflection coefficient on the boundary of medium 1 and medium 2 equals:

$$\Gamma_{1,2} = \frac{\dfrac{Z_2^{\,2}}{Z_3} - Z_1}{\dfrac{Z_2^{\,2}}{Z_3} + Z_1} = \frac{\dfrac{\sqrt{Z_1 Z_3}^{\,2}}{Z_3} - Z_1}{\dfrac{\sqrt{Z_1 Z_3}^{\,2}}{Z_3} + Z_1} = 0$$

In our examples, μ is always the same, so we can just match ε values to the formula, e.g. we set *Frequency* = 100000,00, *Epsilon 1* = 1, *Epsilon 2* = 2, *Epsilon 3* = 4 and *2nd Medium Width in Wavelengths* = 0.25 (Figure 8), and compare it with results obtained for default width value (Figure 7).
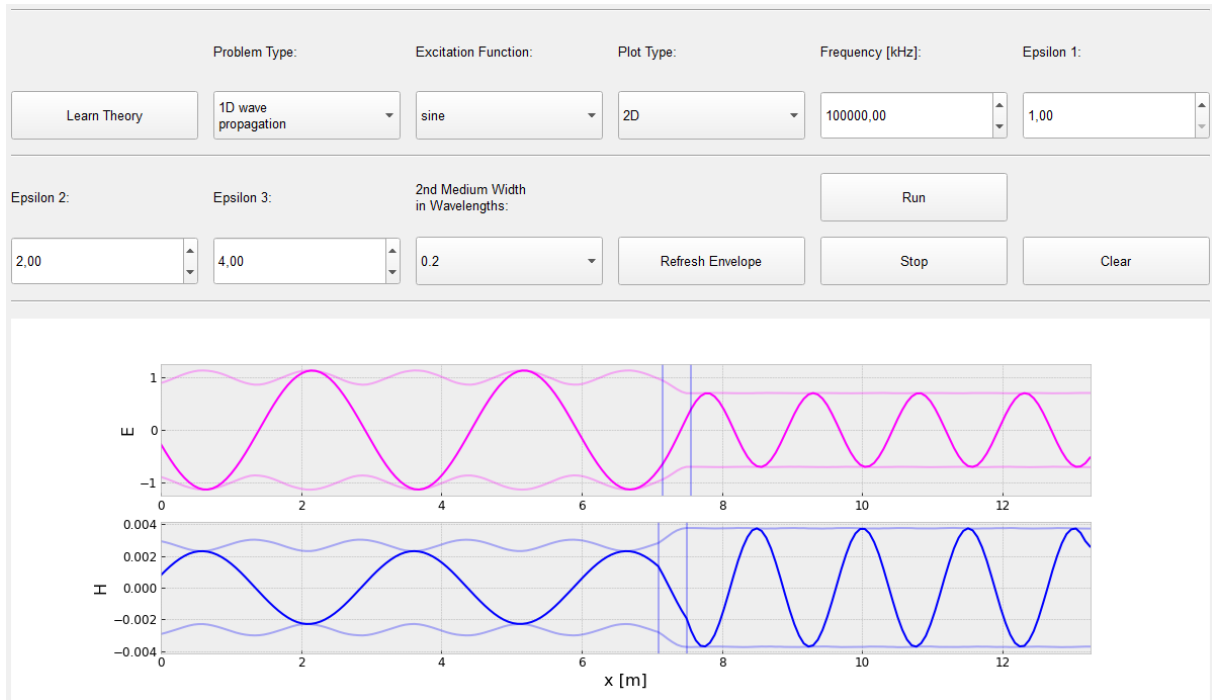

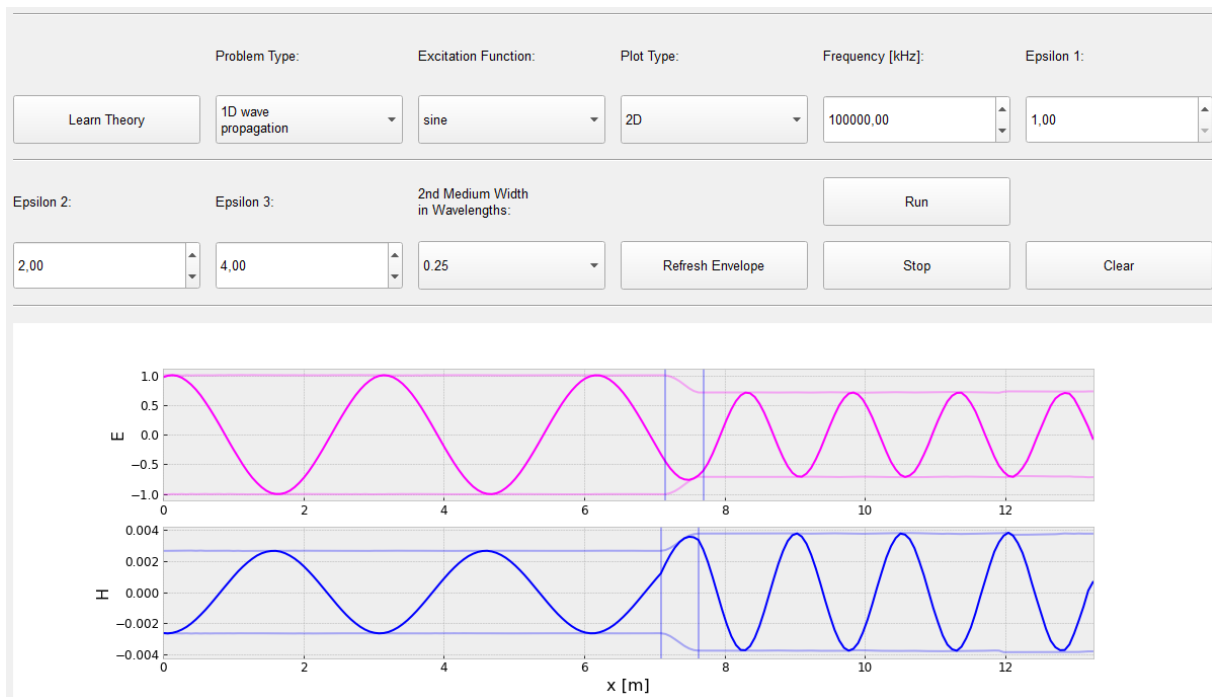*Figure 2.7 Wave propagation – 2nd medium width=0.2*

Figure 2.8 Wave propagation – quarter wavelength transformer, 2nd medium width=0.25

As in the previous example, due to the fact that tangent function is periodic, the thickness of medium 2 can be equal to $\frac{\lambda_2}{4} + n\frac{\lambda_2}{2}$ and the same effect will be obtained (Figure 9).
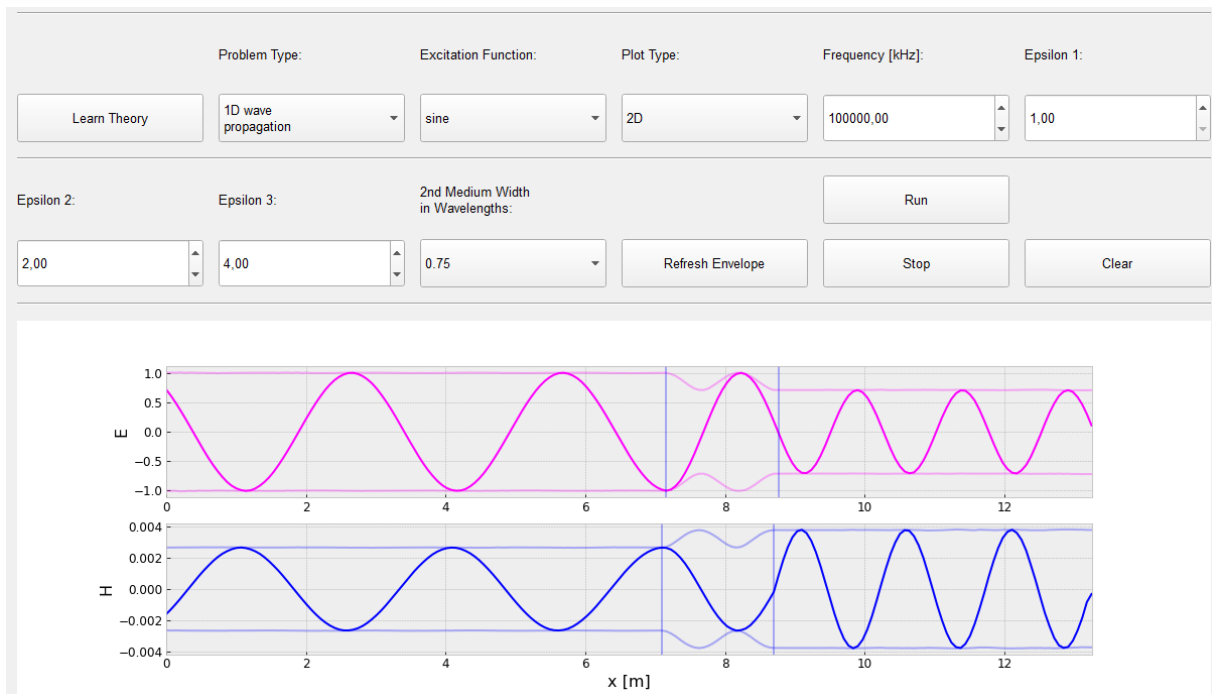
Figure 2.9 Wave propagation – quarter wavelength transformer, 2nd medium width=0.75

1.3. 2D problems

In 2D mode there is a possibility to visualise E-component of electromagnetic wave flowing through only one medium (only *Frequency* and *Epsilon 1* can be changed). This can correspond to an example of infinite conducting rod.

In order to transform our 1D equations into 2D equations we have to consider variations in one additional direction, let us assume this to be Y. In this case, Faraday's law and Ampere's law take the following forms:

$$\frac{\partial Ez}{\partial x} = \mu \frac{\partial Hy}{\partial t}$$

$$\frac{\partial Ez}{\partial y} = -\mu \frac{\partial Hx}{\partial t}$$

$$\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y} = J + \varepsilon \frac{\partial Ez}{\partial t}$$

We can imagine that there is an infinite rod with thickness tending to 0 in the centre of our plane, through which alternating current flows. The equations presented above, mathematically describe this situation and we can observe the graphical representation of this example after running 2D mode simulation.

In order to run a 2D simulation and display a 3D plot, change the *Problem Type* to "2D Wave Propagation" and the *Plot Type* to "3D" (Figure 10).
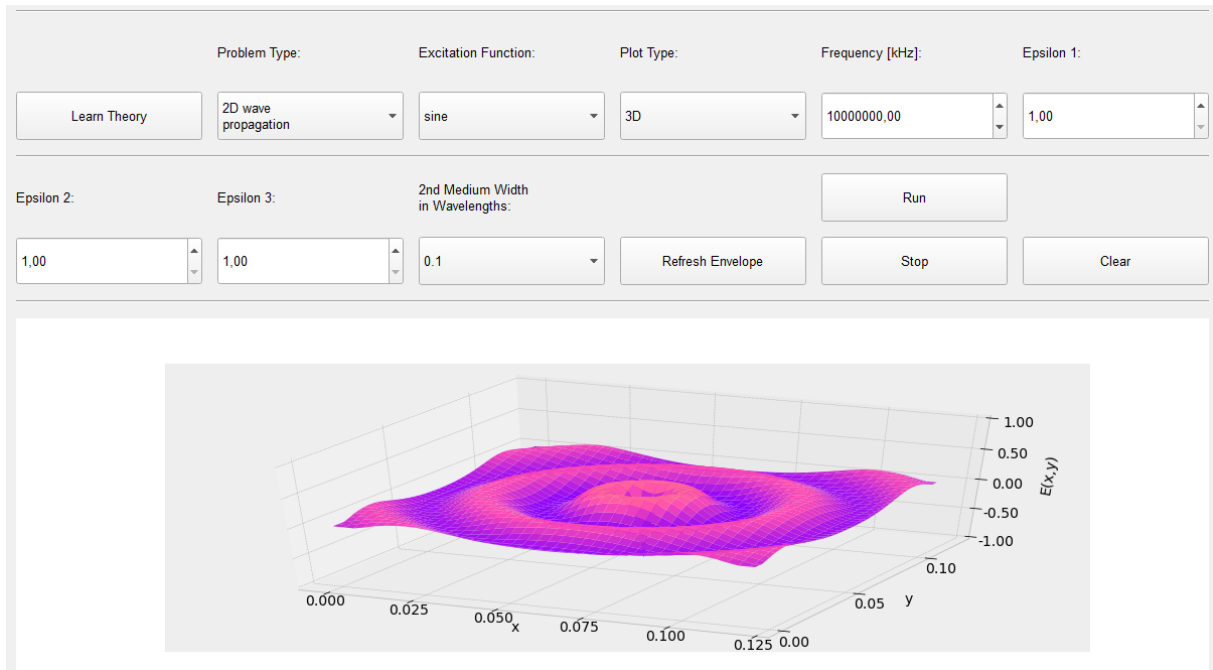
Figure 2.10 2D problem – 3D graphical visualization

Application was created in Python 3.8 using PyQt5 module and matplotlib and numpy libraries.